

# Analysis and measurement of anti-reciprocal systems

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Nov 7, 2014

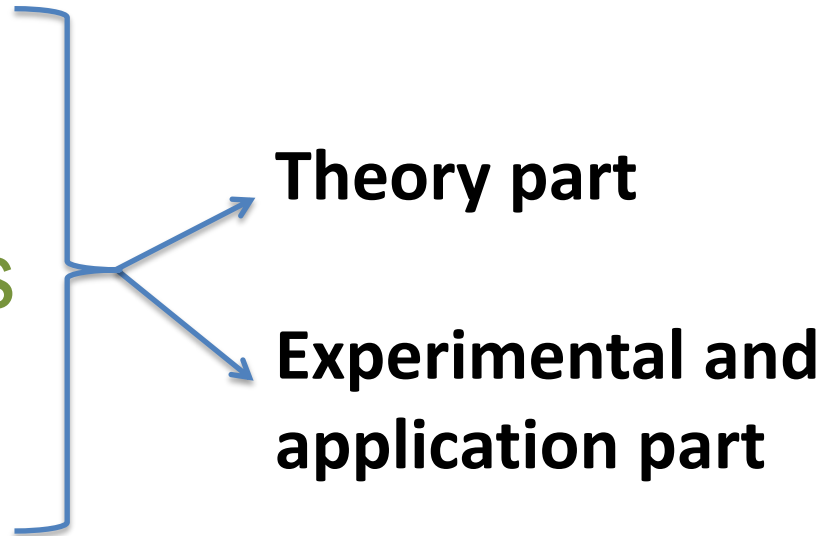


## Thesis objective:

- Provide a clear insight into “**Anti-reciprocal**” systems such as electromagnetic systems

# Thesis contents

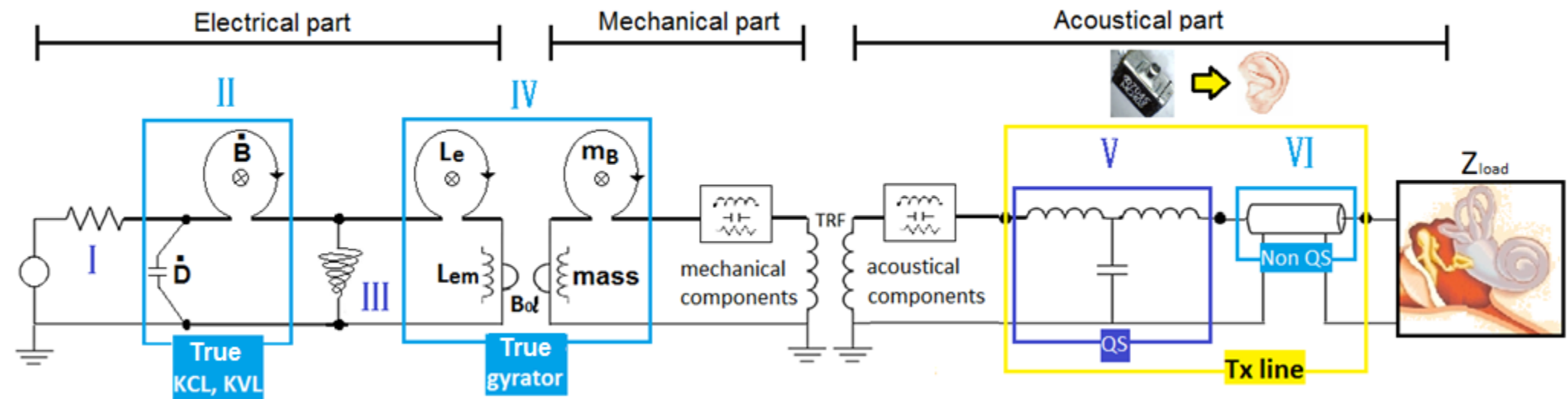
1. INTRODUCTION
2. THEORETICAL METHODS
3. EXPERIMENTAL METHODS
4. RESULTS
5. CONCLUSIONS



Major updates after my preliminary exam....

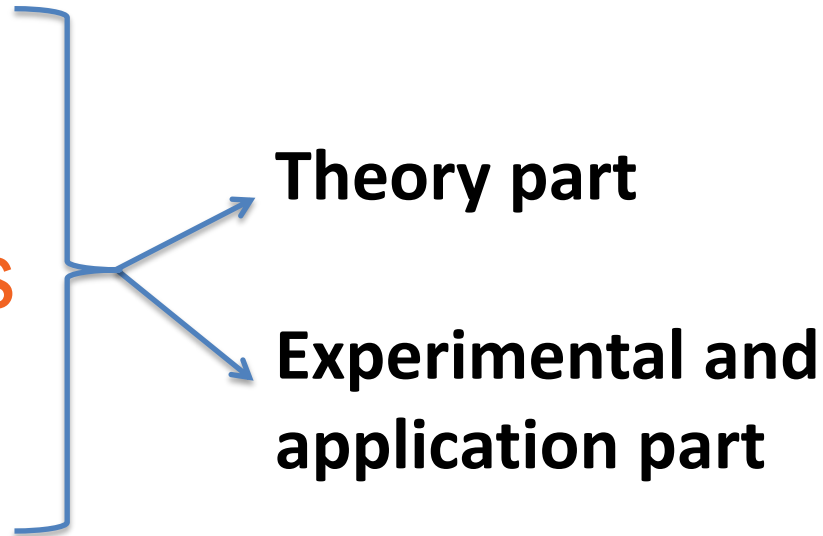
1. Using a conceptual BAR model, I linked all subtopics of my thesis into one place to strengthen and organize my thesis structure

## Projecting thesis topics onto the transducer model



# Thesis contents

1. INTRODUCTION
2. THEORETICAL METHODS
3. EXPERIMENTAL METHODS
4. RESULTS
5. CONCLUSIONS



Major updates after my preliminary exam....

1. Using a conceptual BAR model, I linked all subtopics of my thesis into one place to strengthen and organize my thesis structure
2. I added more experimental work and result, such as hearing measurement probe manufacturing procedure.

## I. Theoretical part

- We take a Balanced Armature Receiver (BAR, a speaker used in hearing-aids) as a specific example of the “**Anti-reciprocal**” system to demonstrate the system’s operational principle

## II. Experimental part

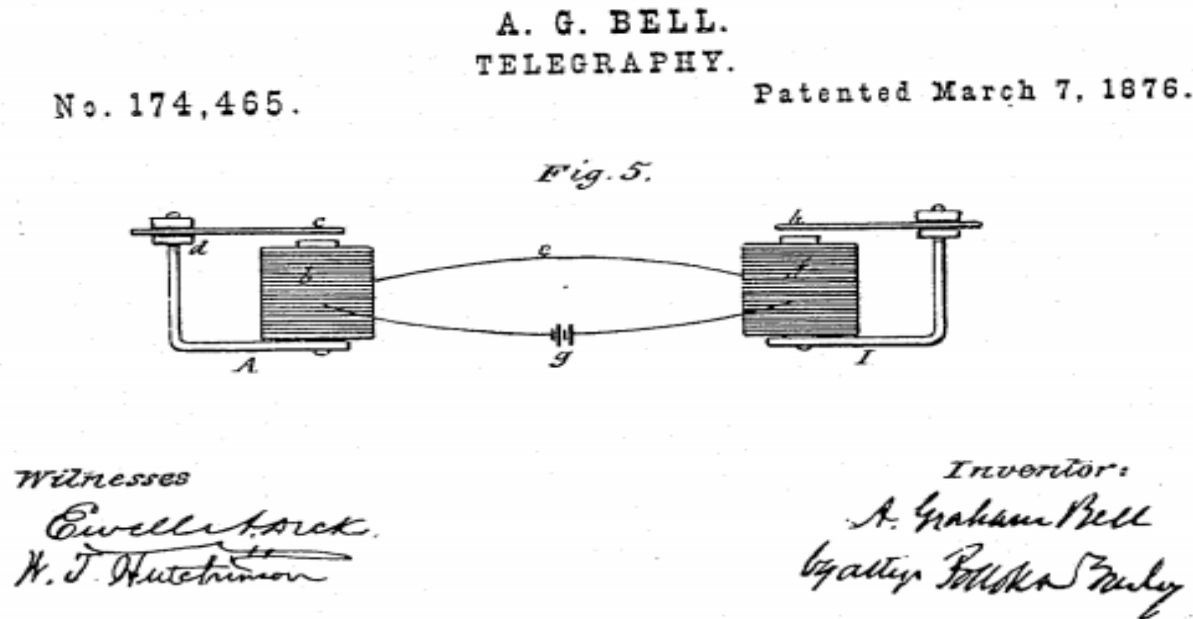
- We introduce
  - Experiments to support (verify) our theory
  - An example to make use of the BAR; the hearing measurement probe manufacturing

# I. Theoretical part

- An answer for the question: how does the BAR work?
  - Introduction of BAR
  - Overview of BAR's operational principle
  - Case study  $I=0$  and  $I\neq 0$  (Eddy-currents)
  - Force on the armature ( $F_m$ ) with Hysteresis effect

# Balanced Armature Receiver (BAR)

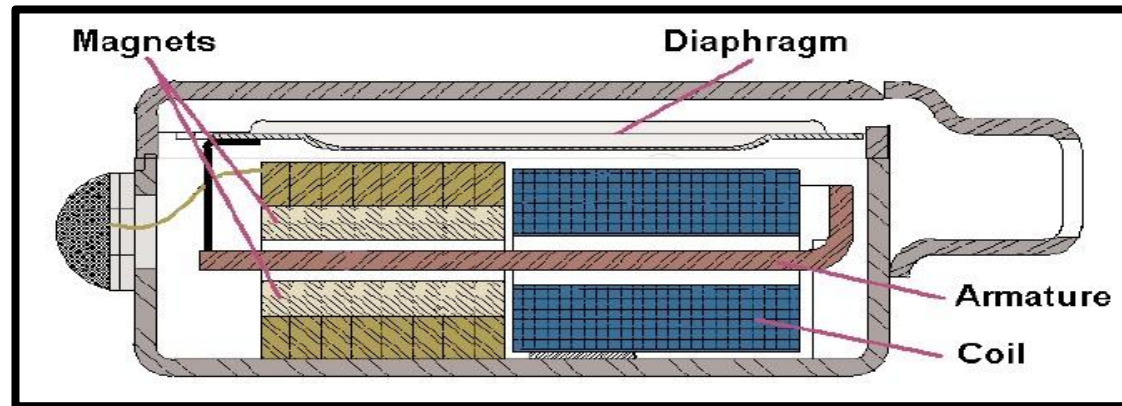
- The oldest telephone receiver was invented by A. G. Bell in 1876



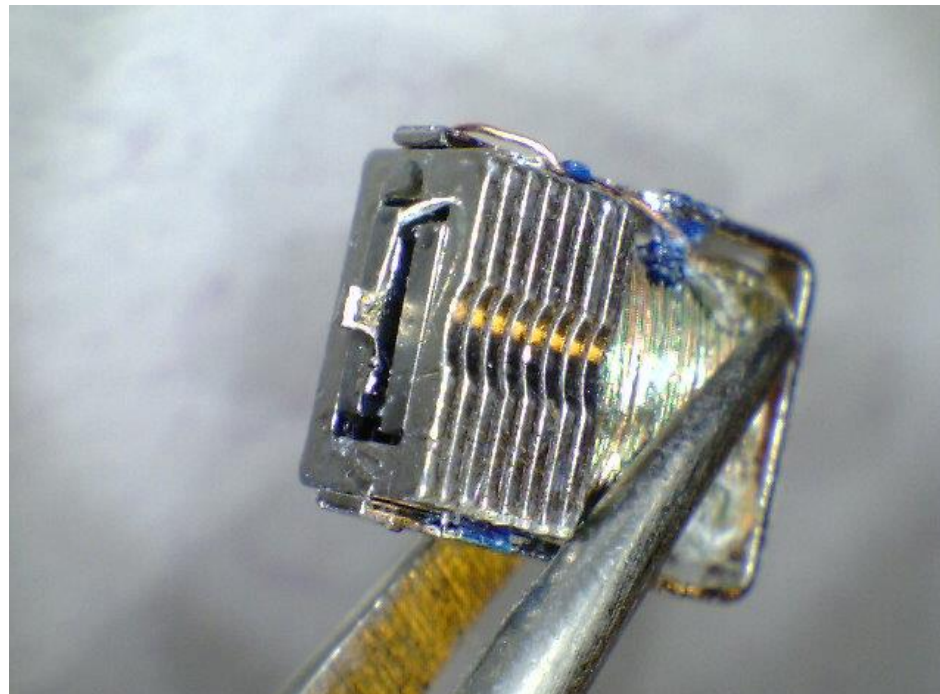
- Attraction and release of the armature are controlled by the current from the coils, which generates electromagnetic fields
- It has evolved into the modern hearing-aid devices



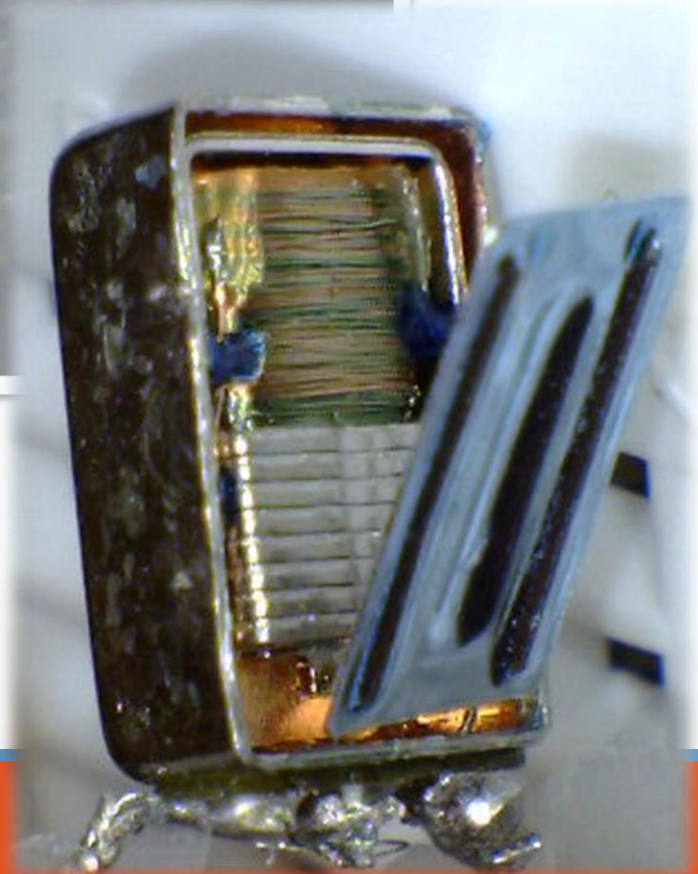
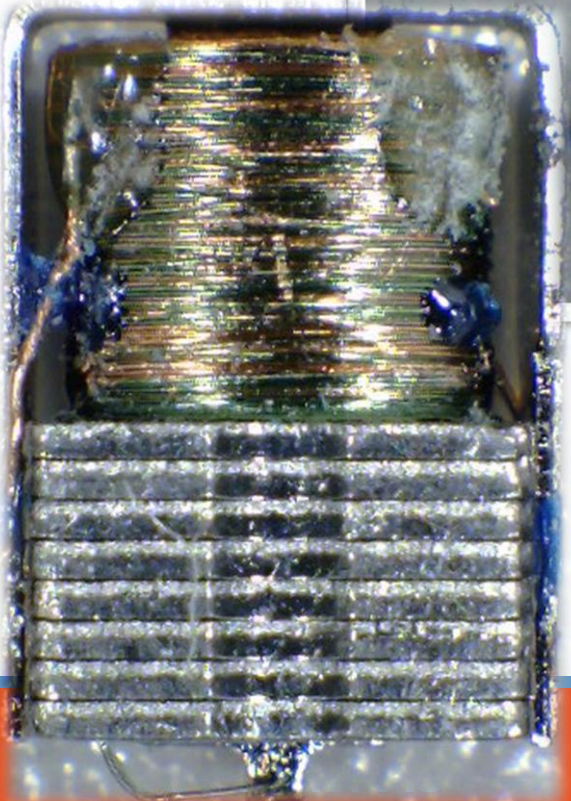
# An example of the modern style BAR, Knowles ED7045



Cross section of Knowles ED receiver



Inside of the BAR without case and diaphragm



# Overview of the BAR's operation

diaphragm

$$\mu_0 \ll \mu_a$$

$\mu_0$

$H$

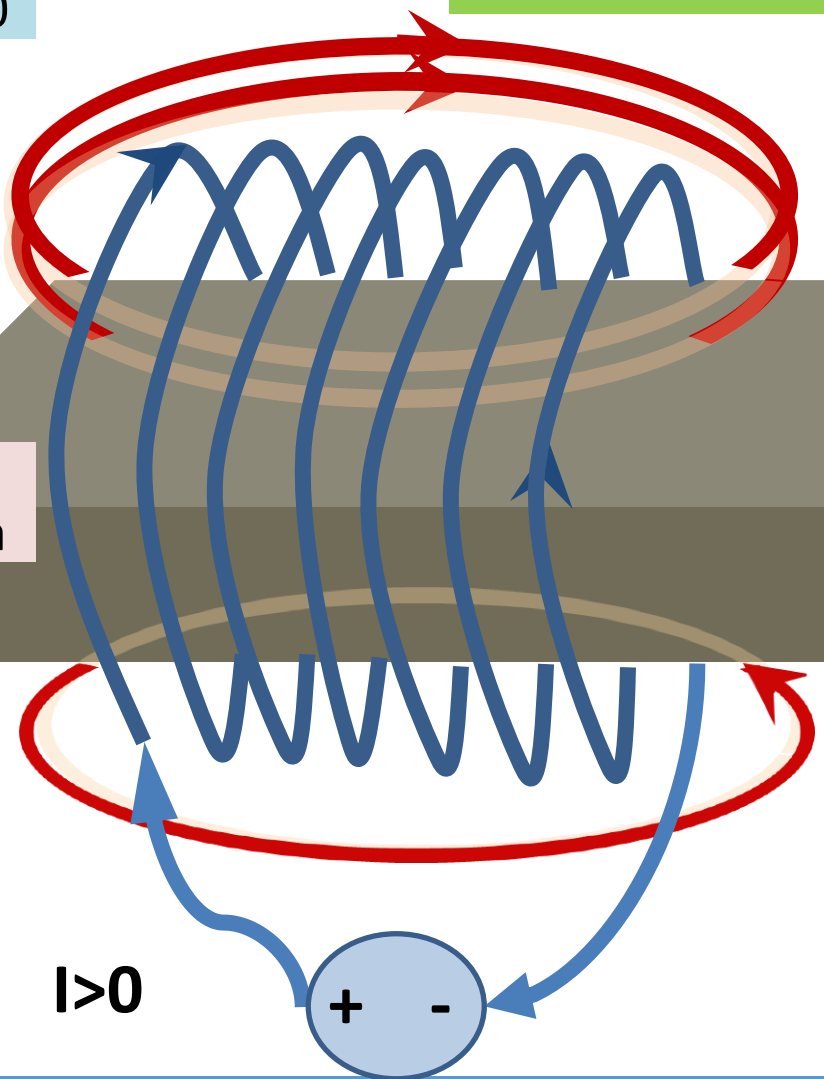
The AC magnetic (solenoid) field's direction is perpendicular to the current.

$\mu_a$

$I > 0$

+

-



diaphragm

$$\mu_0 \ll \mu_a$$

$\mu_0$

$H$

Hysteresis loss (the energy required to rotate the domains of magnetic dipoles) will occur when the induced magnetic field affects the armature.

$\mu_a$

$I > 0$

+ -

diaphragm

$$\mu_0 \ll \mu_a$$

$\mu_0$

$H$

$\mu_a$

$I > 0$

+

-

An eddy current is generated in the opposite direction of the conducting current. This phenomenon is independent of the permanent magnet.

diaphragm

$$\mu_0 \ll \mu_a$$

$\mu_0$

$H$

$\mu_a$

$I > 0$

+

-

S

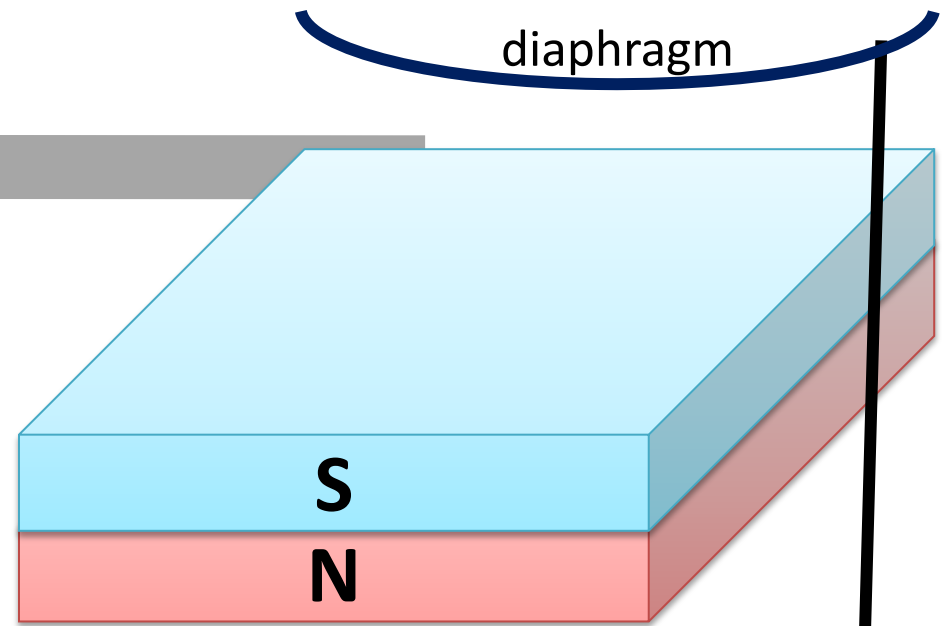
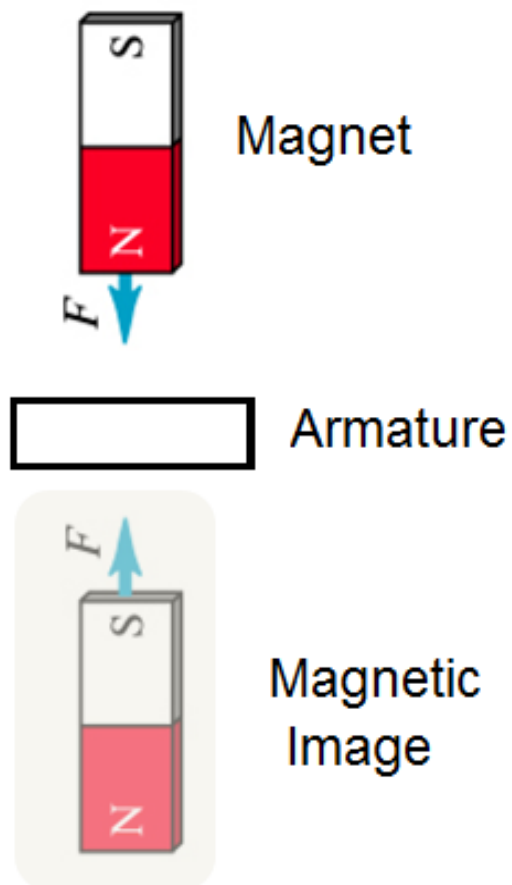
N

S

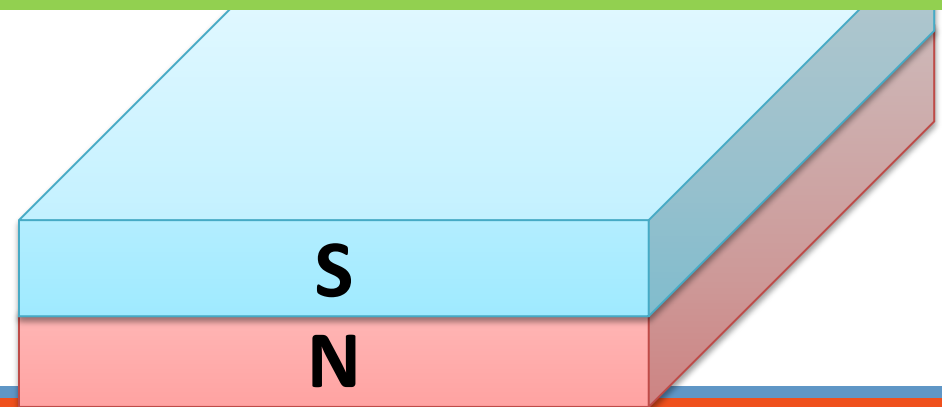
N

Due to the polarity between the permanent (DC) magnetic field and the generated AC magnetic field, the armature feels a force.

Magnetic force,  $F_m$ : Force between two nearby magnetized surfaces to create a magnetic image



Due to the polarity between the permanent (DC) magnetic field and the generated AC magnetic field, the armature feels a force.

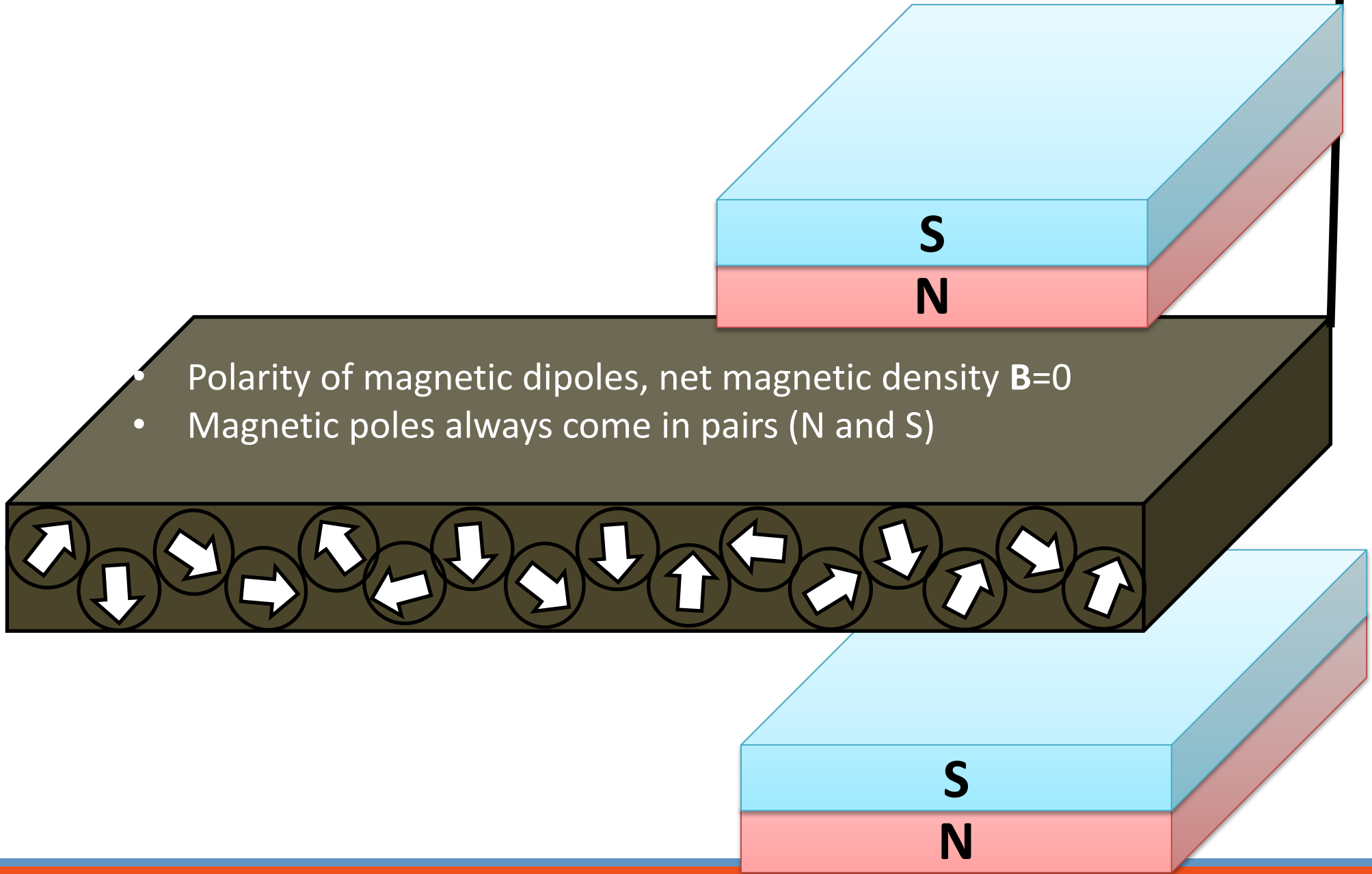




The BAR's behavior:  
 $I = 0$  and  $I \neq 0$  (Eddy-currents)

**I=0**

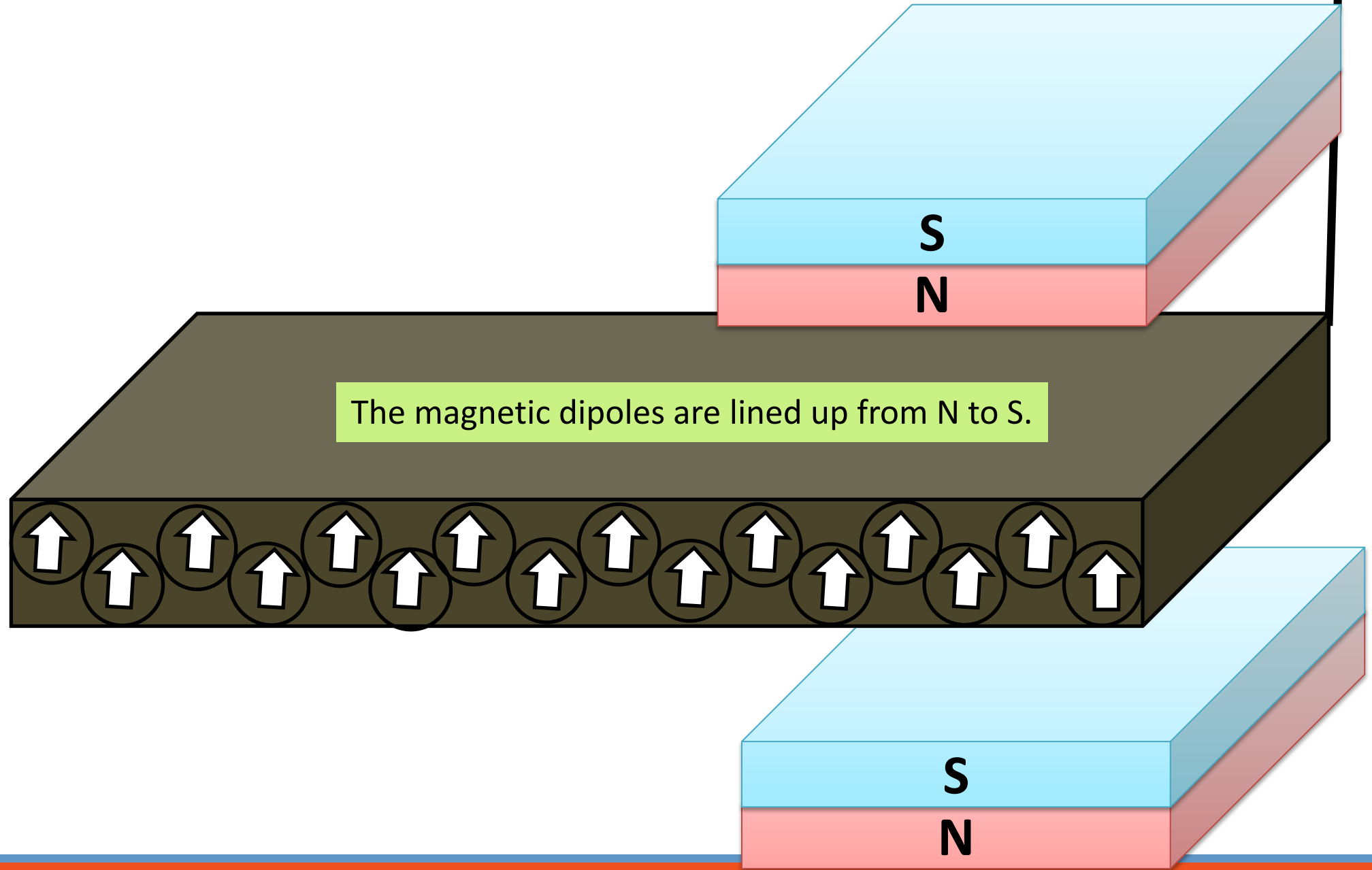
diaphragm



- Polarity of magnetic dipoles, net magnetic density  $B=0$
- Magnetic poles always come in pairs (N and S)

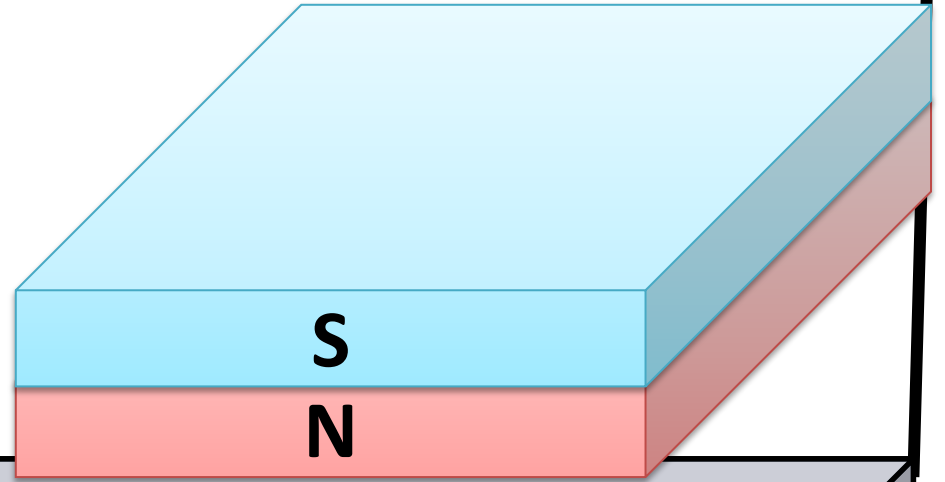
$I=0$

diaphragm

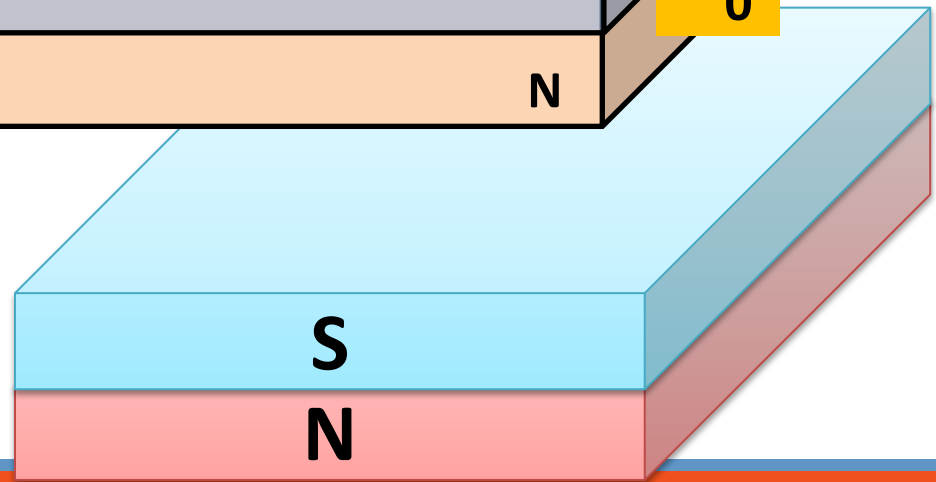
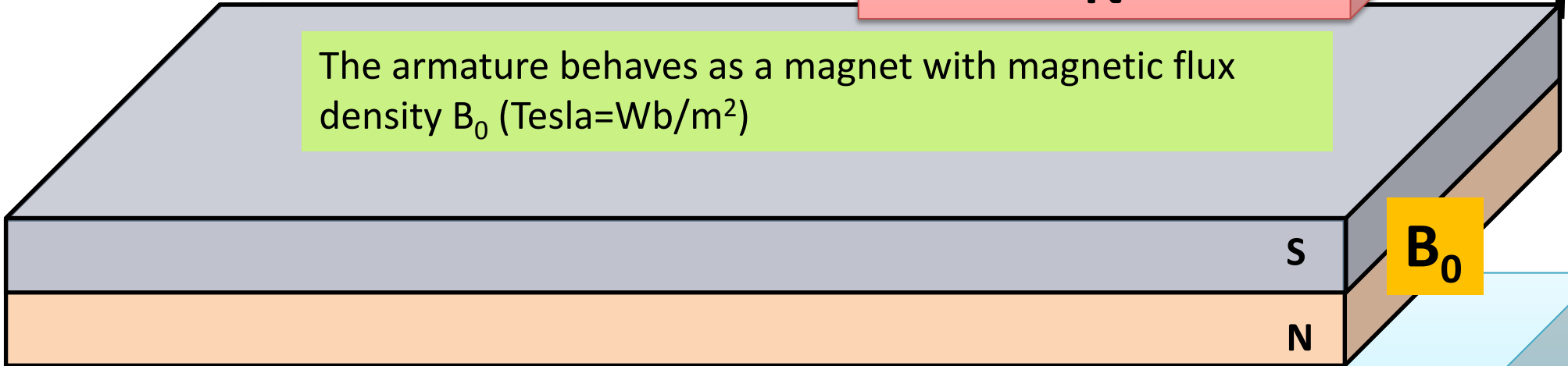


I=0

diaphragm

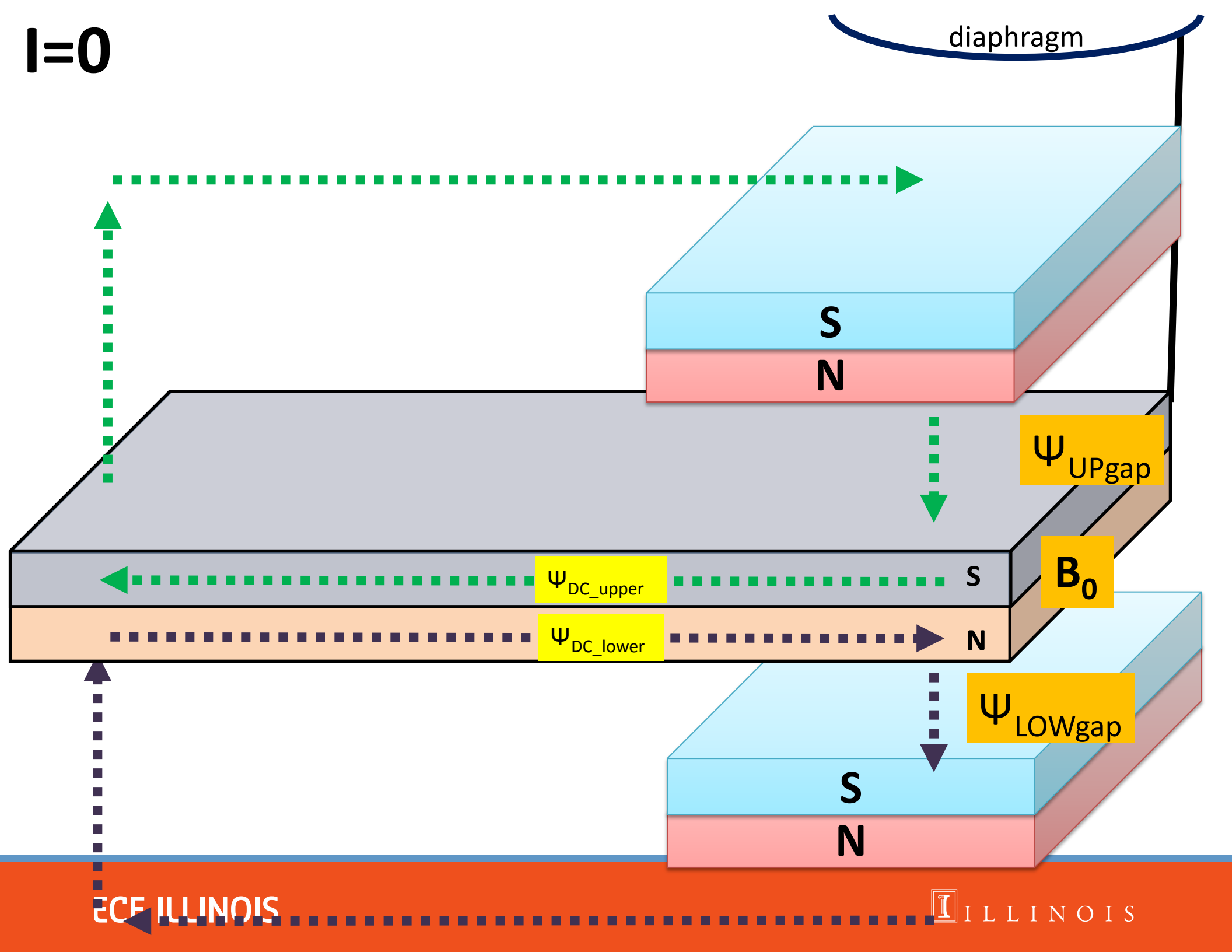


The armature behaves as a magnet with magnetic flux density  $B_0$  (Tesla=Wb/m<sup>2</sup>)



I=0

diaphragm



I=0

diaphragm

The armature is balanced

- $\psi_{DC\_upper} + \psi_{DC\_lower} = 0$  (the net flux)

$\uparrow F_{UPgap}$

$\psi_{UPgap}$

$\psi_{DC\_upper}$

S

$B_0$

$\psi_{DC\_lower}$

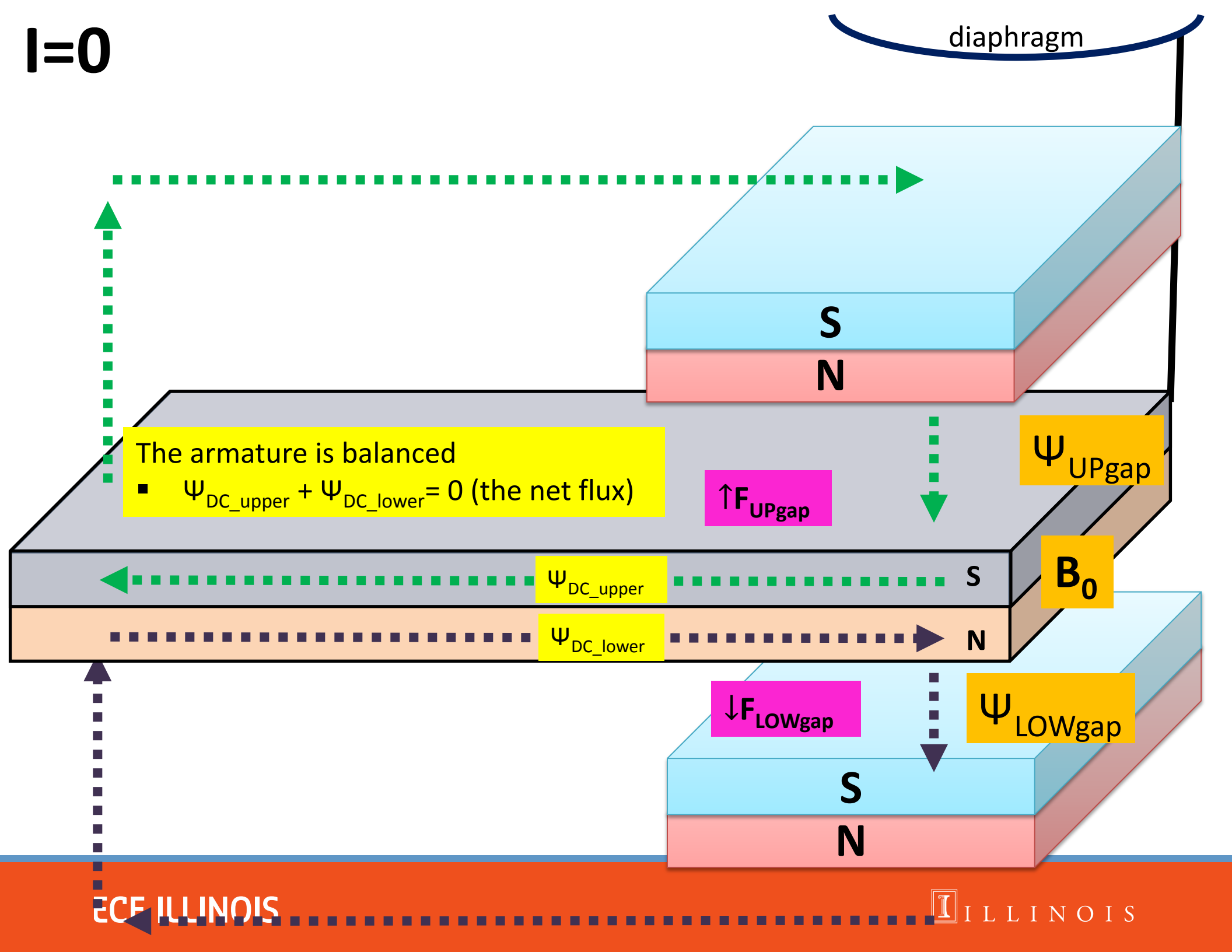
N

$\psi_{LOWgap}$

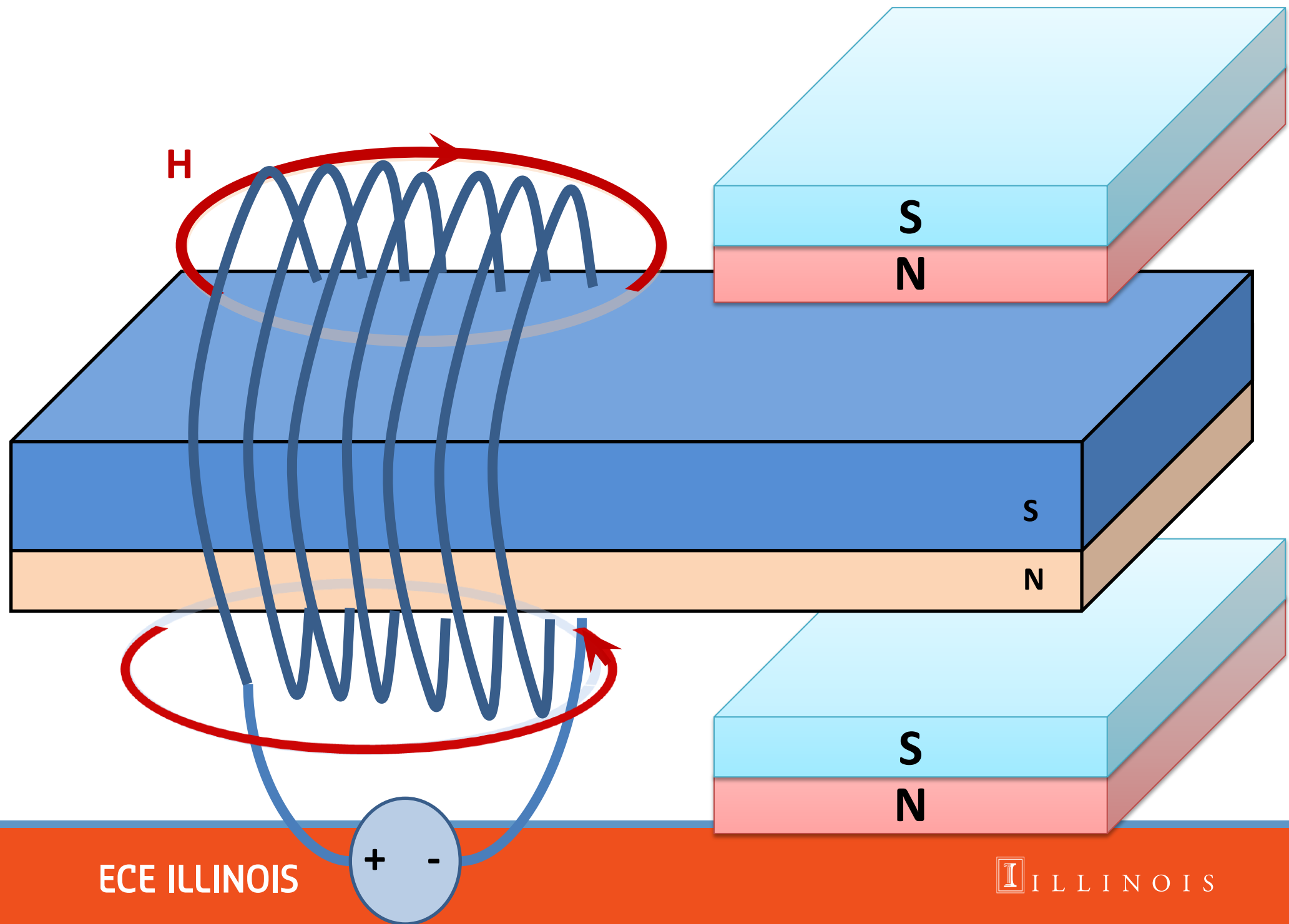
$\downarrow F_{LOWgap}$

S

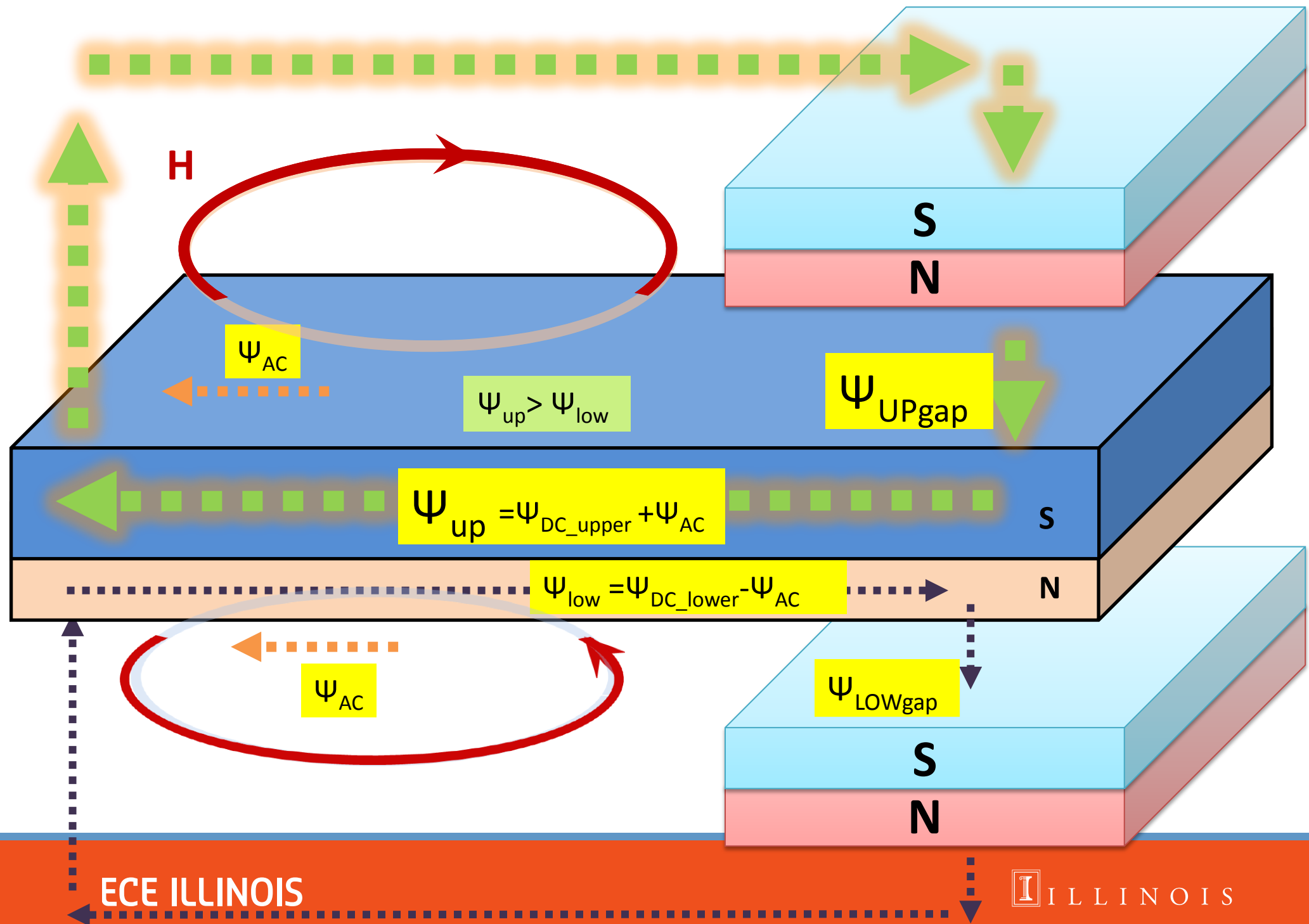
N



$I > 0$



$I > 0$



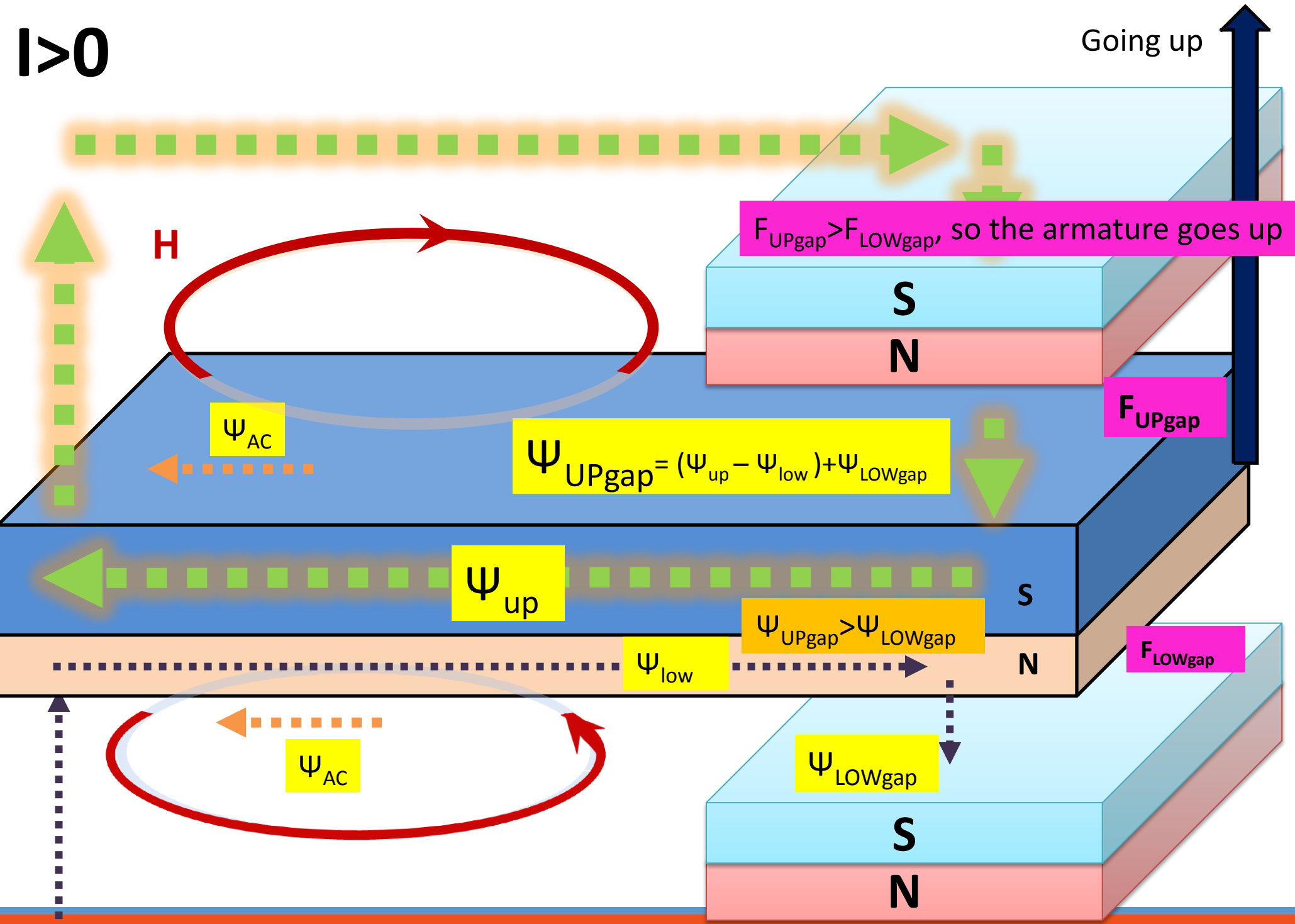


$I > 0$

Going up

H

$F_{UPgap} > F_{LOWgap}$ , so the armature goes up

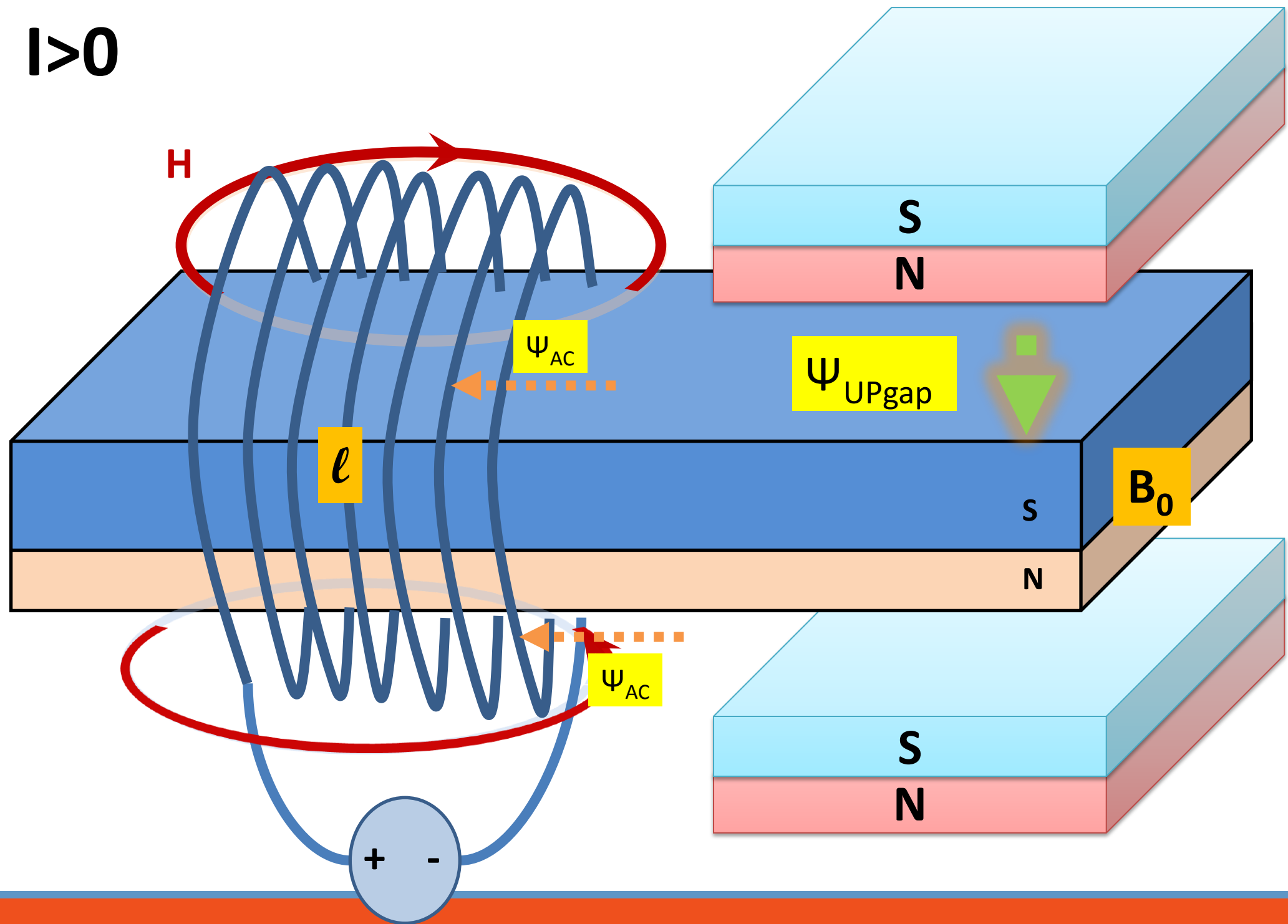


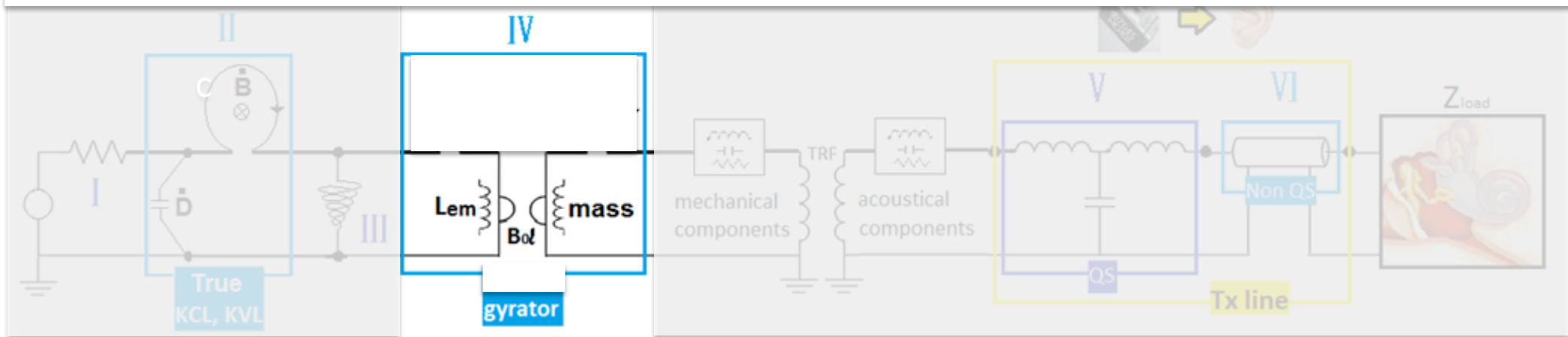
- A gyrator swaps the generalized flow and force (Impedance matrix)

$$\begin{bmatrix} \Phi(\omega) \\ F(\omega) \end{bmatrix} = \begin{bmatrix} 0 & -B_0 l \\ B_0 l & 0 \end{bmatrix} \begin{bmatrix} I(\omega) \\ U(\omega) \end{bmatrix} \Rightarrow \begin{array}{l} \text{Eq.1} \\ \Phi(\omega) = -B_0 l U(\omega) \end{array} \text{ and } \begin{array}{l} \text{Eq.2} \\ F(\omega) = B_0 l I(\omega) \end{array}$$

Two Eqs. for an Ideal gyrator

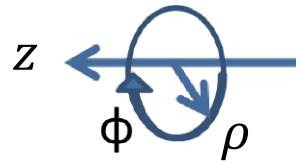
$I > 0$





# Eddy current

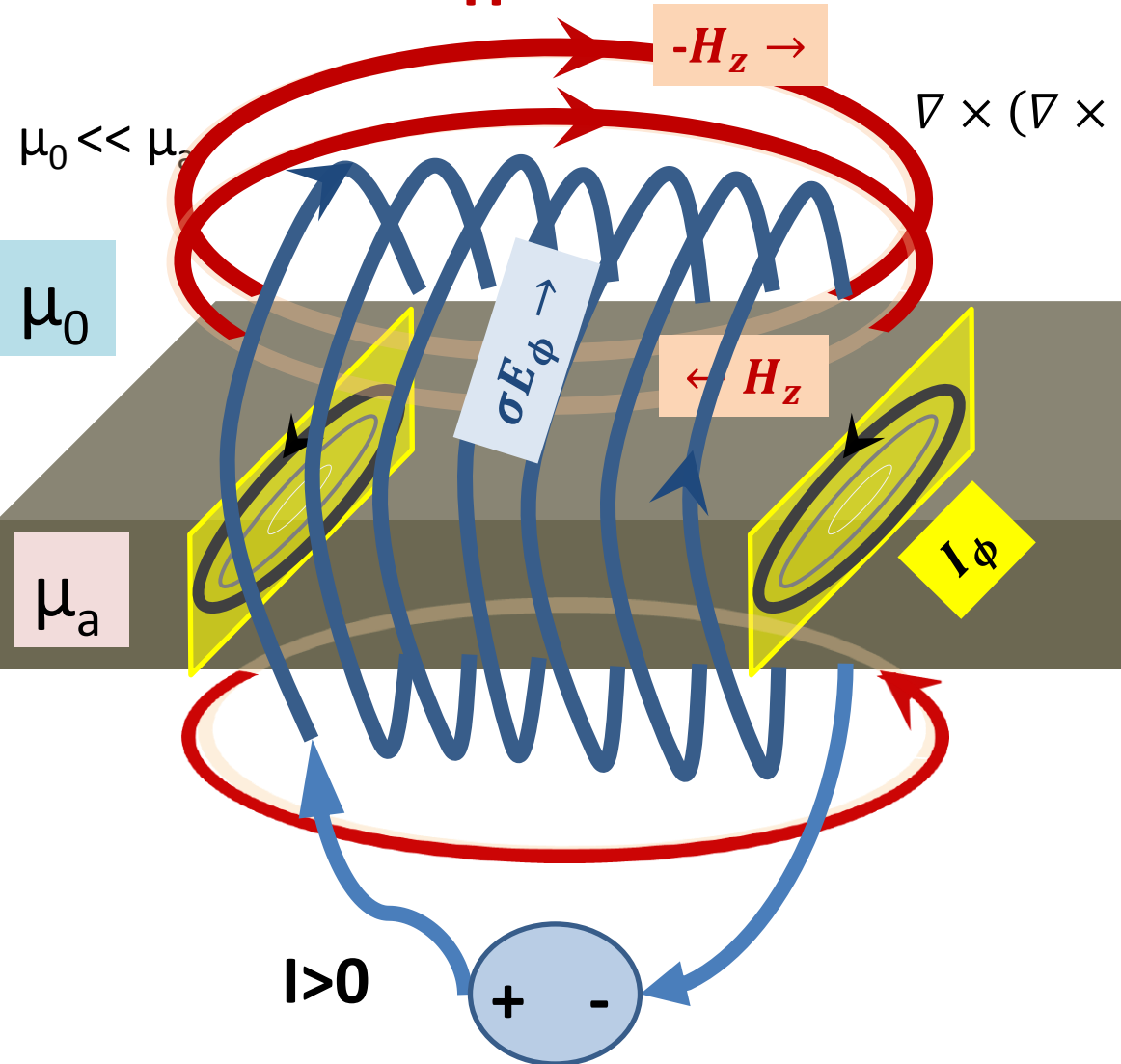
(Vanderkooy 1989)



$$\nabla \times \mathbf{H}_z = J_{c\phi} + \dot{\mathbf{D}} \approx J_{c\phi} = \sigma \mathbf{E}_\phi \quad (1. \text{ Ampere's law})$$

$$\nabla \times \mathbf{E}_\phi = -\dot{\mathbf{B}}_z \quad (2. \text{ Faraday's law})$$

$$\nabla \times (\nabla \times \mathbf{H}) = \nabla(\underbrace{\nabla \cdot \mathbf{H}}_0) - \nabla^2 \mathbf{H} \quad (3. \text{ Vector identity})$$



$$\nabla \times (\nabla \times \mathbf{H}_z) = -\nabla^2 \mathbf{H}_z \quad (::3)$$

$$\nabla \times (\sigma \mathbf{E}_\phi) = -\nabla^2 \mathbf{H}_z \quad (::1)$$

$$\sigma \nabla \times \mathbf{E}_\phi = -\sigma \dot{\mathbf{B}}_z \quad (::2)$$

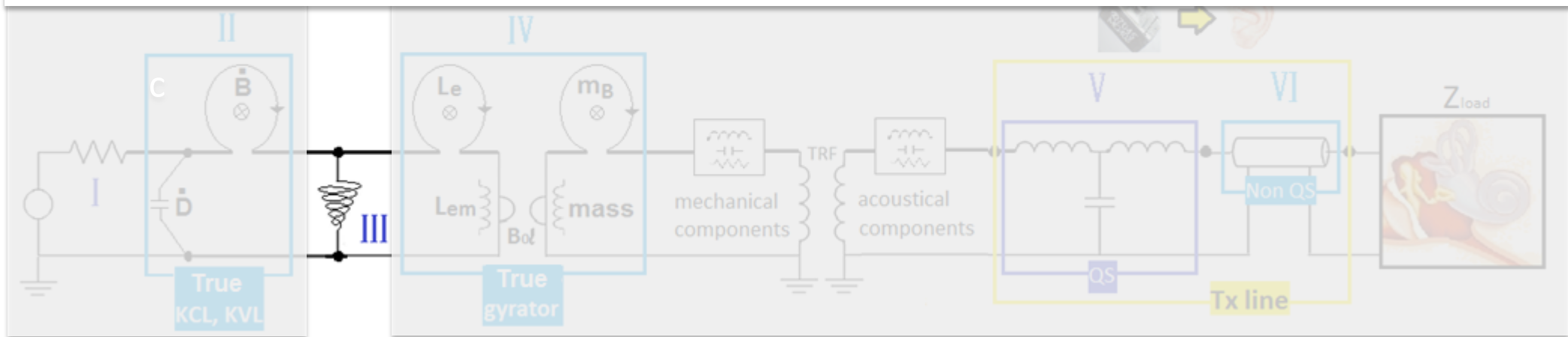
Finally,  $\nabla^2 \mathbf{H}_z = \sigma \mu_a \frac{d\mathbf{H}_z}{dt}$

In the frequency domain

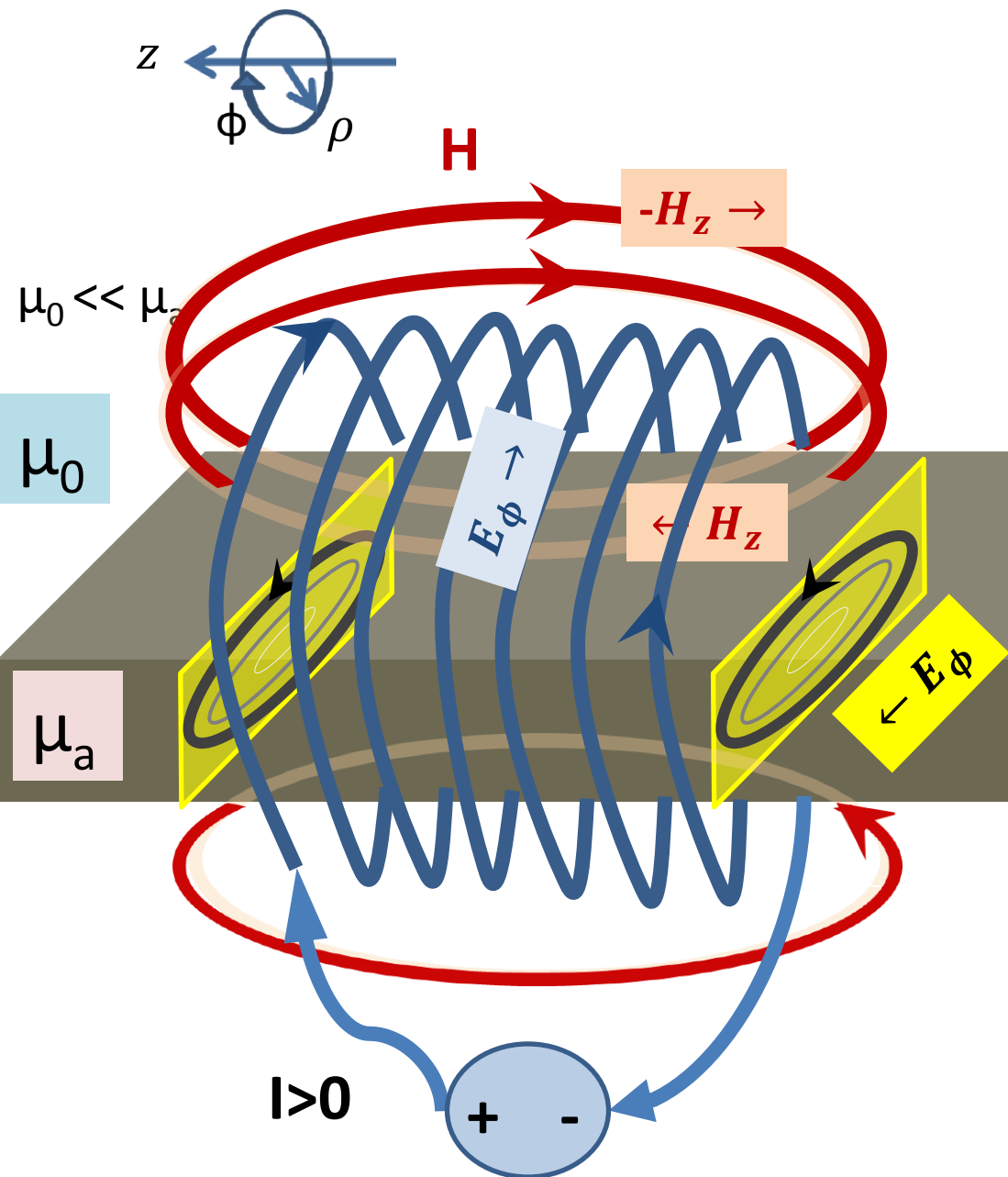
$$(jk)^2 = \sigma \mu_a j \omega$$

$$k_\rho = \pm \sqrt{\sigma \mu_a \omega} e^{-\angle 45^\circ} \quad (\text{diffusion})$$

$$2\mathbf{H}_z(\rho, t) = 2\mathbf{H}_0 e^{j\omega t - k\rho}$$



# Eddy current



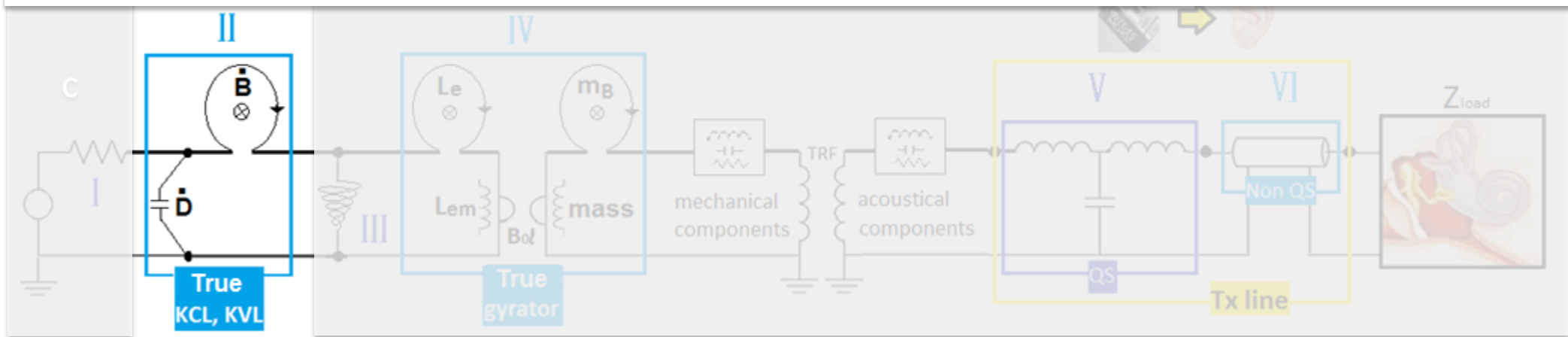
Manipulating the Faraday's law,

$$emf = \int \mathbf{E}_\phi \cdot d\mathbf{l} = \int \nabla \times \mathbf{E}_\phi \cdot d\mathbf{A}$$

$$= - \int \dot{\mathbf{B}}_z \cdot d\mathbf{A} = -\dot{\Psi}_a$$

where,  $dA$  is the cross sectional area of the armature core.

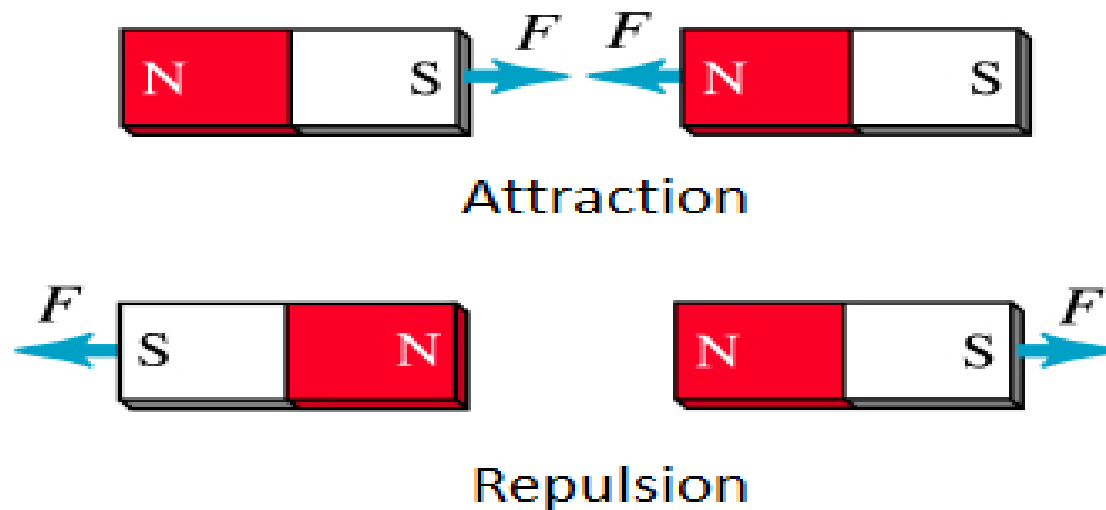
- Emf is Thevenin voltage (true KVL)





# Force on the armature and hysteresis

- Force on the armature ( $F_m$ ) exists for two opposing poles across an air gap
  - Opposite poles attract and like poles repel



- **Hysteresis** can be explained by describing the  $F_m$ ,

- Assumption: Core is initially not magnetized

1. Electrical energy:  $W = \int v(t)i(t)dt$  [J = N·m]

2.  $W_d = \int \frac{HlAdB}{lA} = \int HdB$  Faraday:  $B = \frac{\phi}{A}$  Ampere:  $H = \frac{I}{l}$   $\left[ \frac{J}{m^3} = \frac{N}{m^2} \right]$

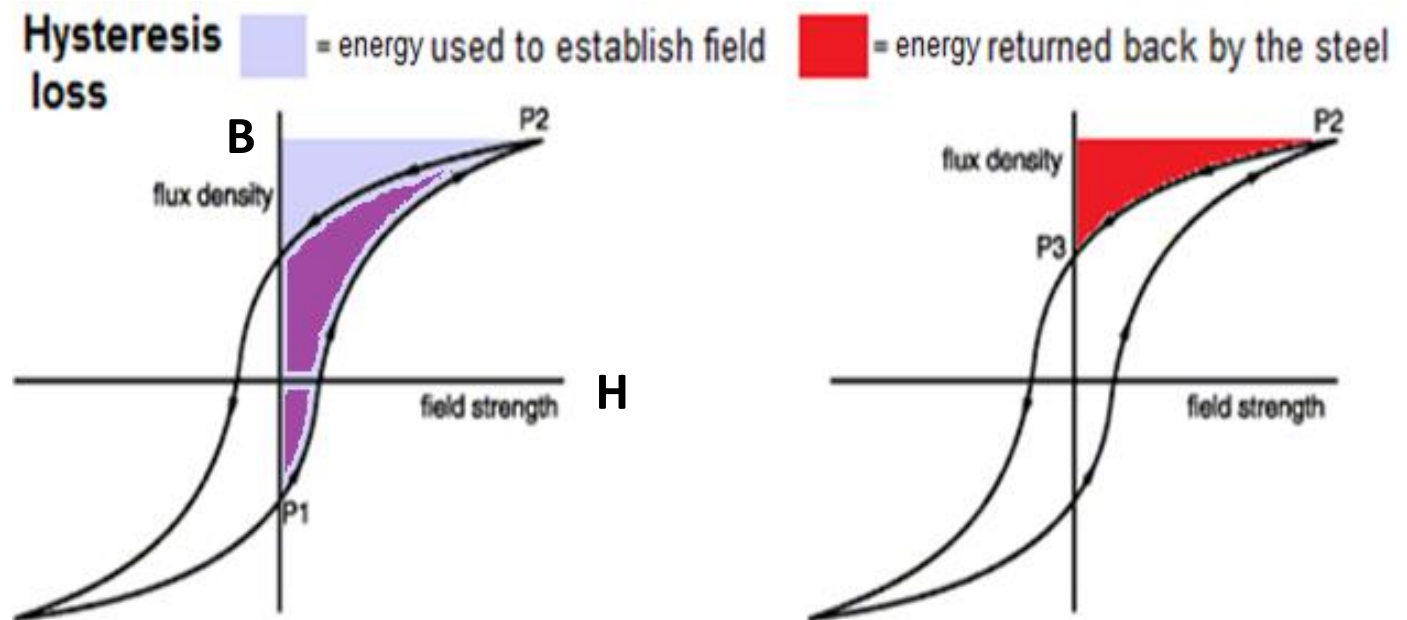
3. Therefore  $F_m = W_d A$

$$F_m = \frac{AB^2}{2\mu} = \frac{A_g B_g^2}{2\mu_0} = \frac{\Psi_g^2}{2\mu_0 A_g} [N]$$

$$W_d = \int \frac{HlAdB}{lA} = \int \mathbf{HdB} = \frac{1}{\mu} \int BdB = \frac{B^2}{2\mu} \left[ \frac{J}{m^3} = \frac{N}{m^2} \right]$$

The green formula can be related to the famous hysteresis loop graph

- x-axis and y-axis represent H and B
- Hysteresis loss: subtraction of two regions
- A typical hysteresis phenomenon of Ferro-magnetic material



([http://info.ee.surrey.ac.uk/Workshop/advice/coils/power\\_loss.html#eddy](http://info.ee.surrey.ac.uk/Workshop/advice/coils/power_loss.html#eddy))

*We are interested in BAR's operational region*

- Hunt 1954, Ch. 7, Moving armature transducer systems
- BAR type receivers are operating in a lens shaped region

- The region can be linearly approximated
- Centered at  $\Psi_0$  (due to the permanent magnet)
- Alternating  $\Psi_i$

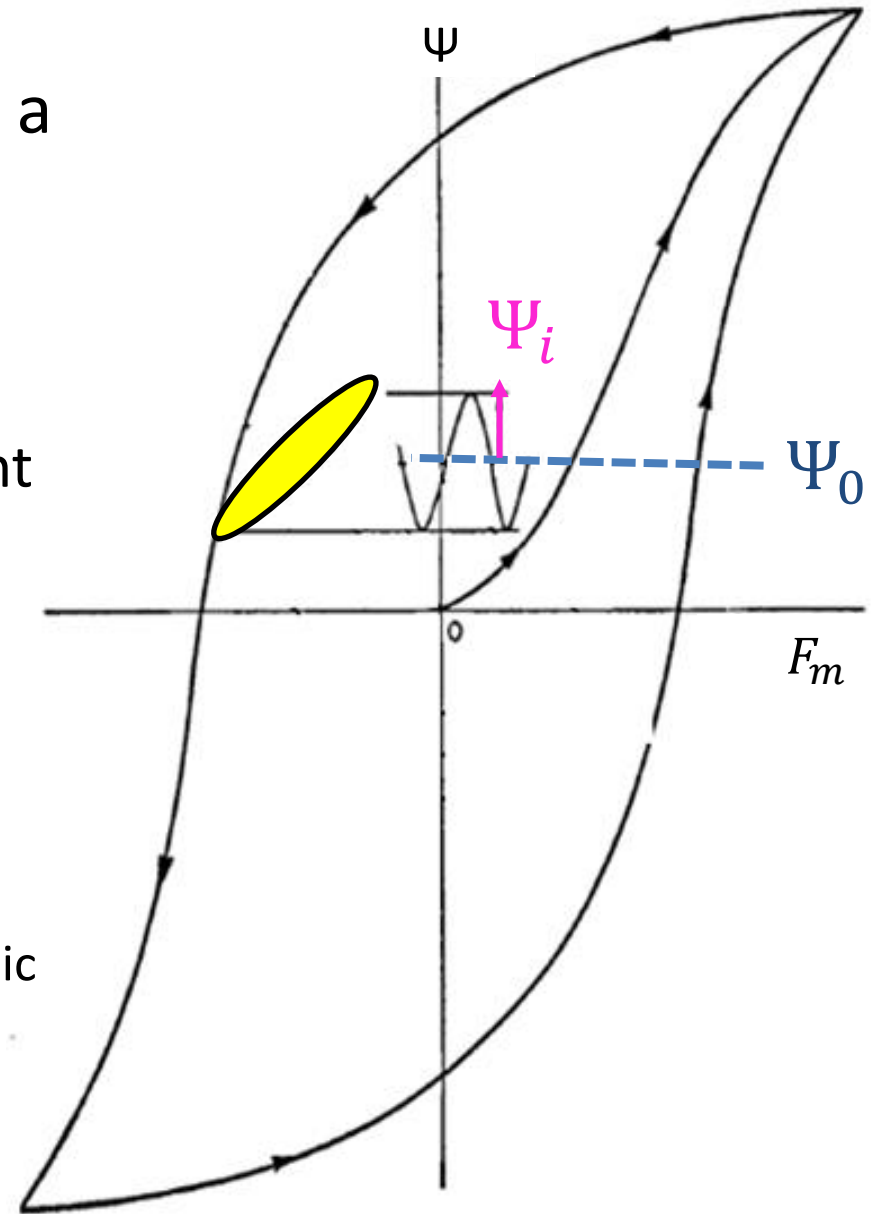
$$F_m = \frac{\Psi_g^2}{2\mu_0 A_g} = \frac{(\Psi_0 + \Psi_i)^2}{2\mu_0 A_g}$$

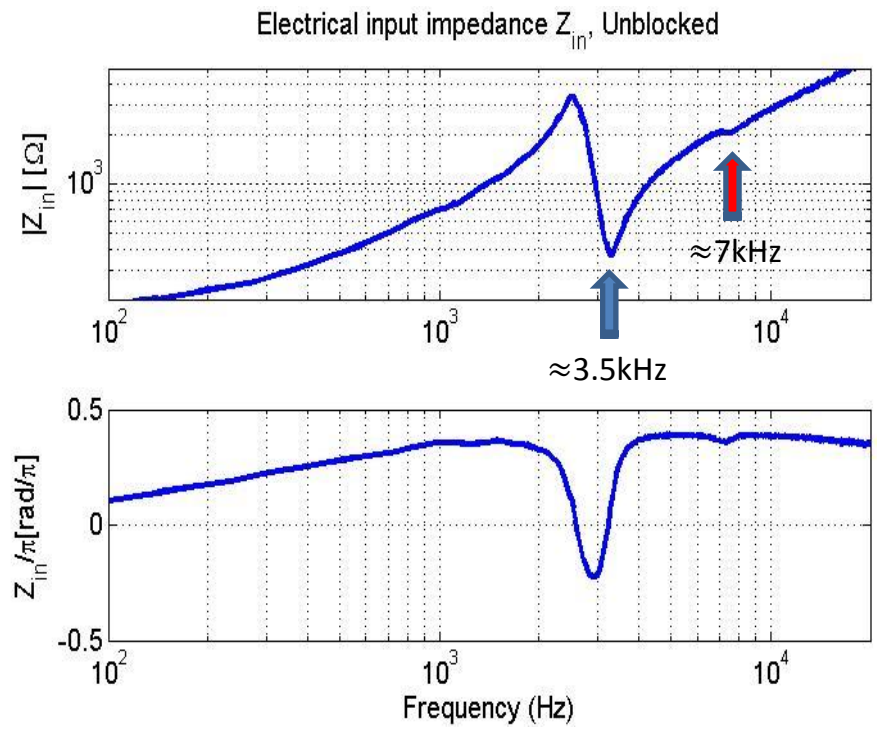
$$= \frac{\Psi_0^2 + 2\Psi_0\Psi_i + \Psi_i^2}{2\mu_0 A_g}$$

→ Non-linear part  
Second harmonic distortion

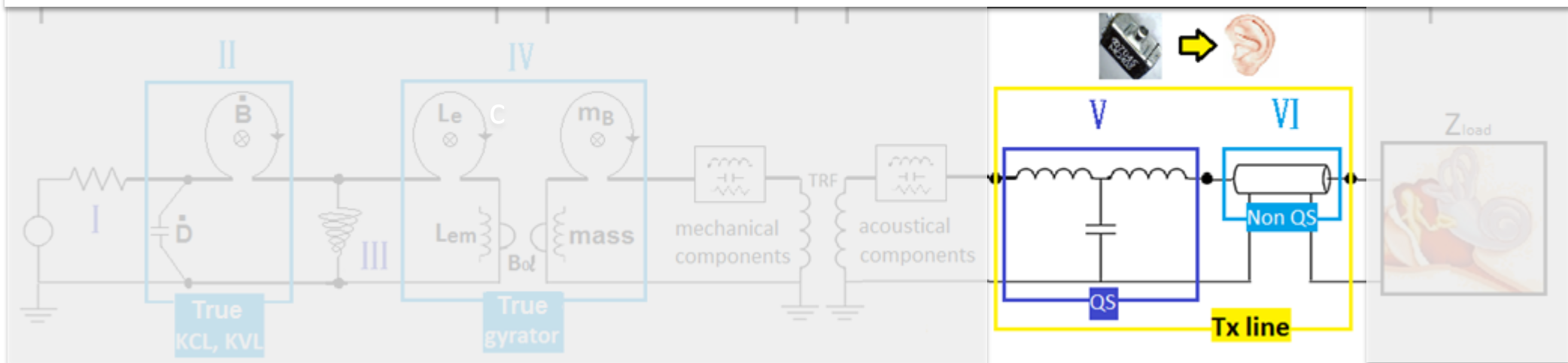
- If  $\Psi_i = \Psi_I \cos \omega t$ , then

$$\Psi_i^2 = \frac{1}{2} \Psi_I^2 (1 + \cos 2\omega t)$$





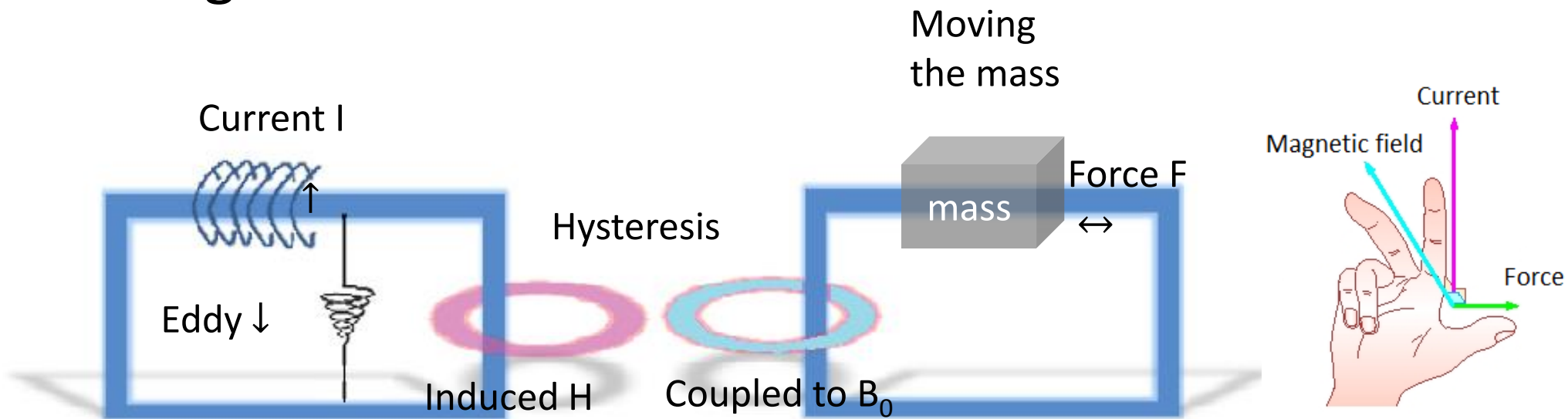
# Quasi-static (QS) and delay



- Let's discuss this topic at the end of this presentation 😊

# Sub conclusion from theory part

- Principles of the BAR's operation include the Eddy-current effect, hysteresis loss, and force on the two magnets



- This work will provide a fundamental, clearer insight into this type of BAR system

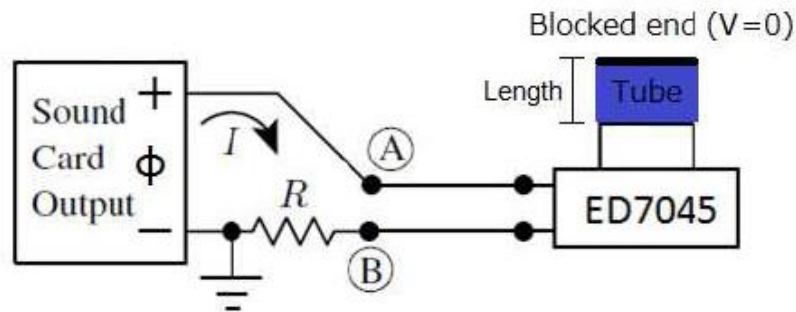


## II. Experimental part

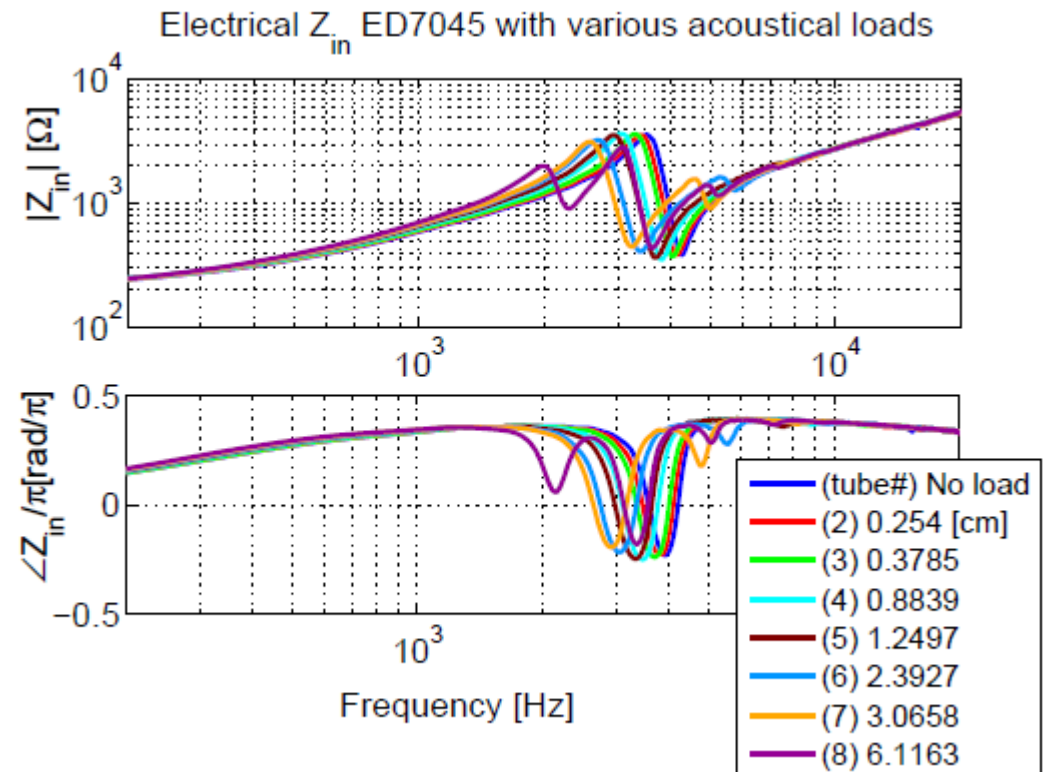
- Experiments to support (verify) our theory
  - Electrical input impedance measurements
  - Laser vacuum measurements
  - Pressure measurements
- Hearing measurement probe manufacturing
  - Existing probe study
  - Manufacturing and evaluation

# Experiments to support our theory

# Electrical input impedance measurements

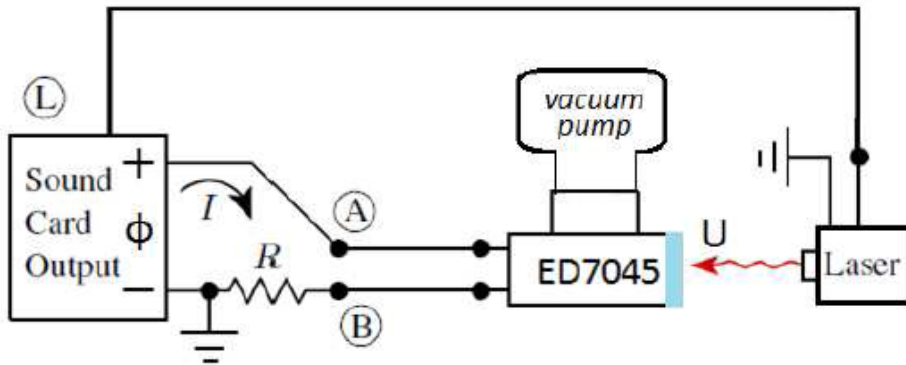


$$Z_{in} = \frac{\Phi_A - \Phi_B}{I} = \frac{\Phi_A - \Phi_B}{\Phi_B/R} = R \left( \frac{\Phi_A}{\Phi_B} - 1 \right).$$



- Used for the Hunt parameter calculation

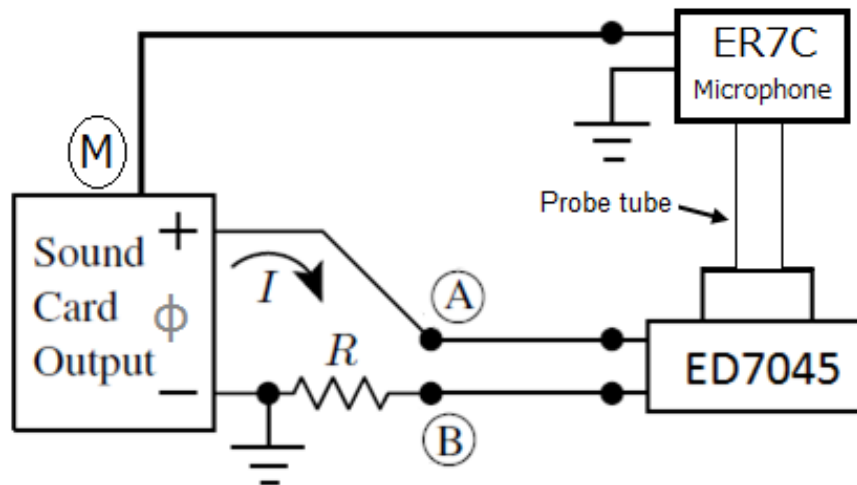
## Laser vacuum measurements



- A portion of the transducer's case was carefully removed
- Then a thin plastic window was glued on, to reseal the case



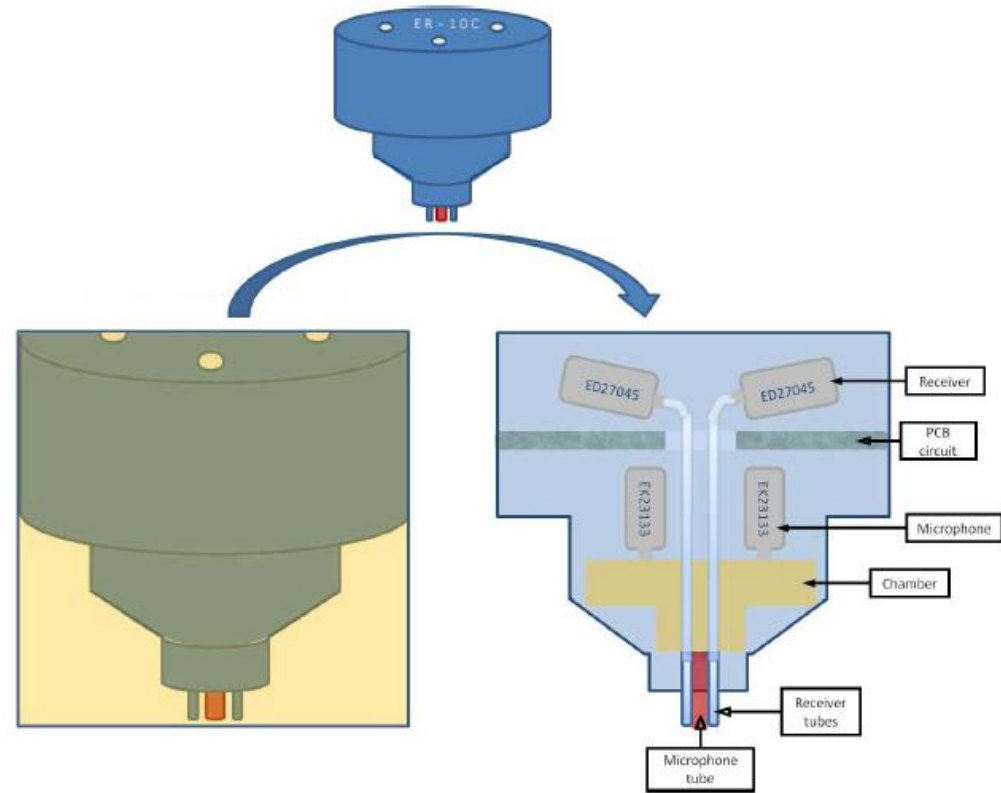
## Pressure measurements



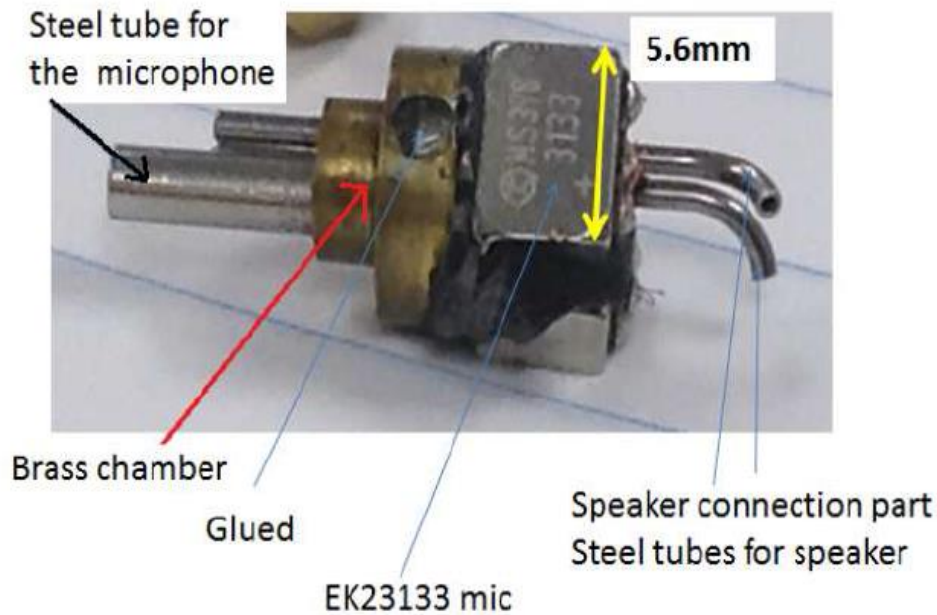
- The circled 'M' means an input from the ER7C microphone

# Hearing measurement probe manufacturing

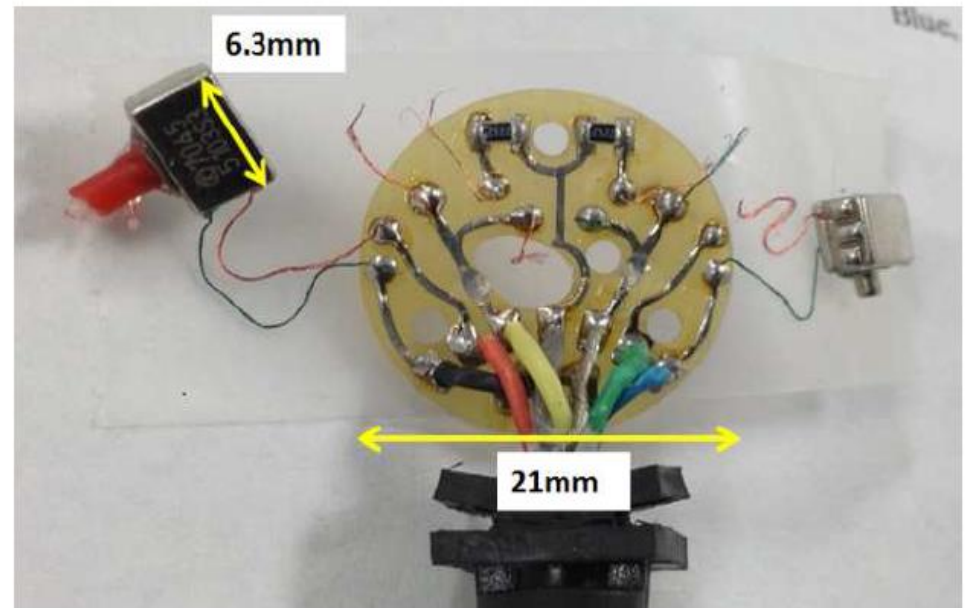
# Existing probe study: ER10C (Etymotic Research)



- Otoacoustic emission (OAE, sounds given off by the inner ear when the cochlea is stimulated by a sound) measurement device
- Two speakers and microphones are separated internally across the PCB circuit, microphones are placed ahead of the receivers

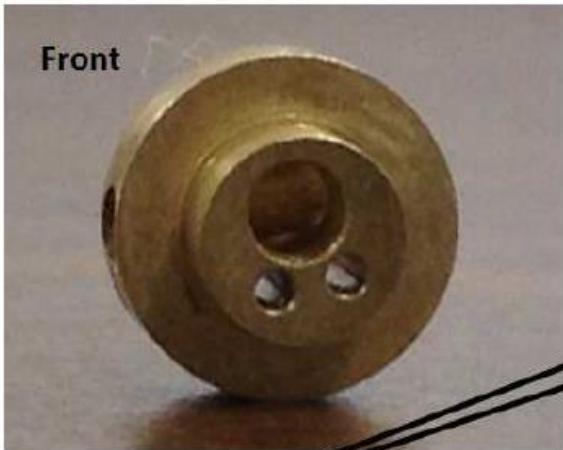


(a) ER10C microphone holder

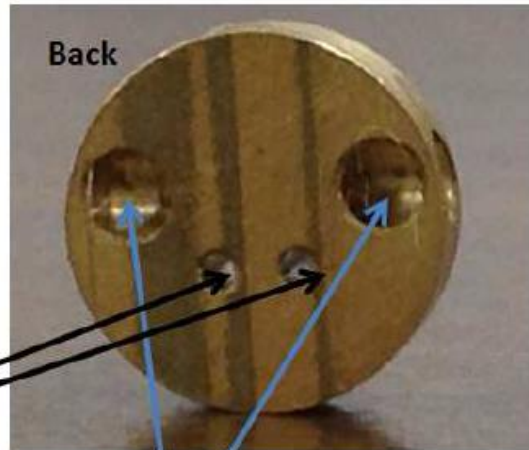


(b) ER10C circuit board part

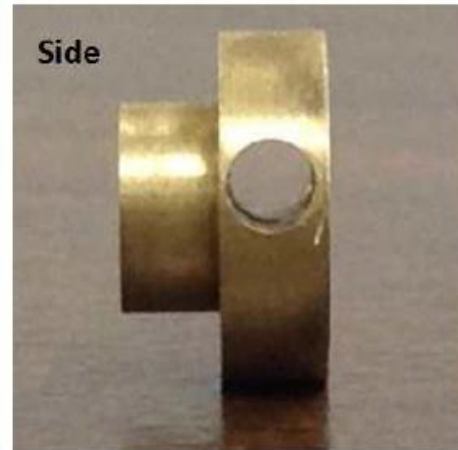
- The microphones are firmly attached to the chamber
- The speakers are attached to steel tubes via a soft rubber tubes, floated in the air
  - The air is a best damper, vibrational crosstalk from the speakers can be reduced



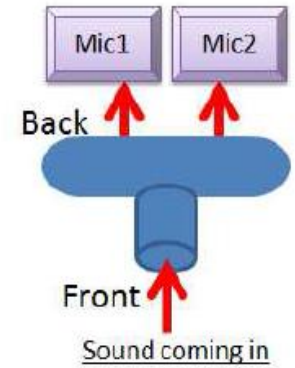
Front



Back



Side

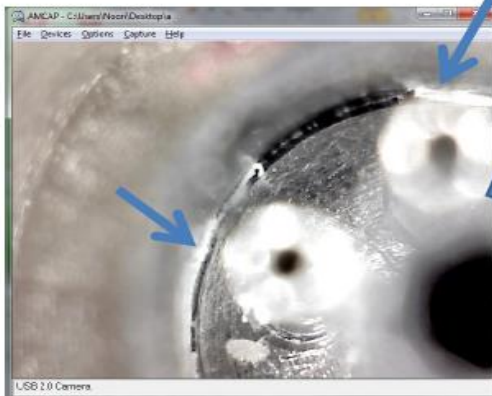


The microphone's sound path is cylindrical 'T' shape

ED27045 speaker's thin steel tube holder

EK23133 microphone port holder

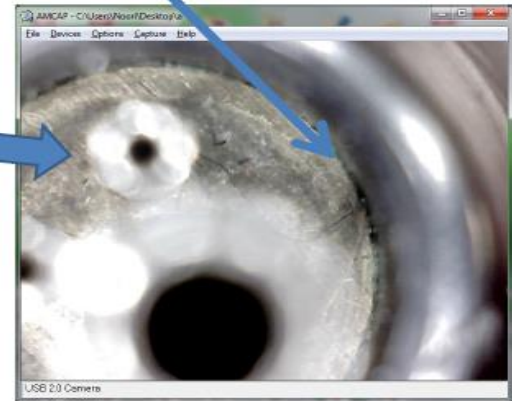
ER10C New design, #2972  
Gap, no glue, the white jammed thing is putty



Looking down from the ER10C probe front with its case



ER10C Old design, #465  
The middle part fits well to the case of ER10C. RTV could be applied to the edge area

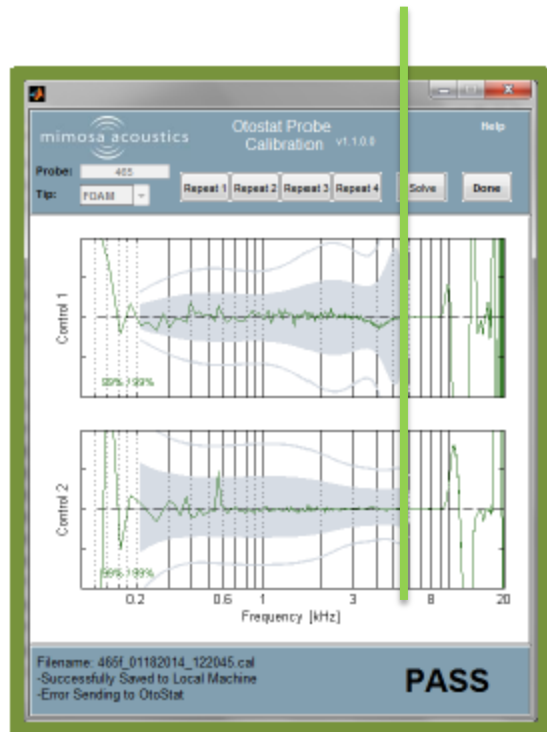




## Issues with ER10C

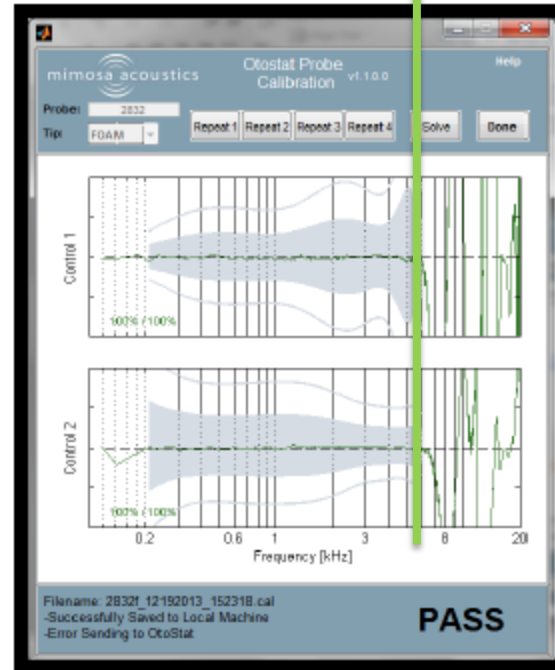
- The small number of competitors in the market, users have not had many alternatives to the system
- The size of the probe is too big for infants
- Handling it without extreme caution may lead to malfunction of the probe (delicate device)
- The result of the measurement depends too much on the condition of the foam tip that is inserted in the subject's ear canal
- Above 6 kHz, calibration (always) fails (**the most critical problem**)

# Problem: above 6 kHz, calibration fails



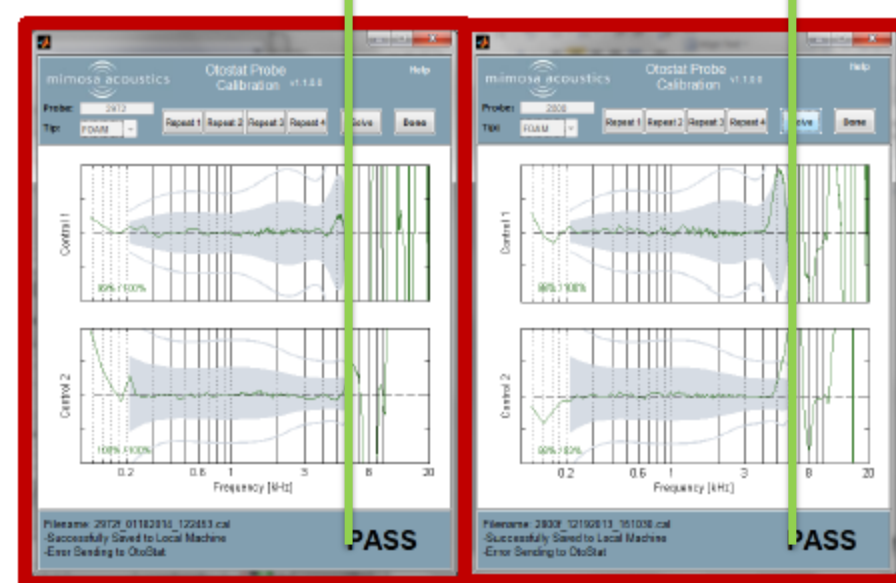
## OLD ER10C

- Brass material for the middle tube holder part
- RTV is used to block the holder's side hole
- Calibration passes up to 9-10kHz
- ER10C with 3 digits serial number



## New Good ER10C

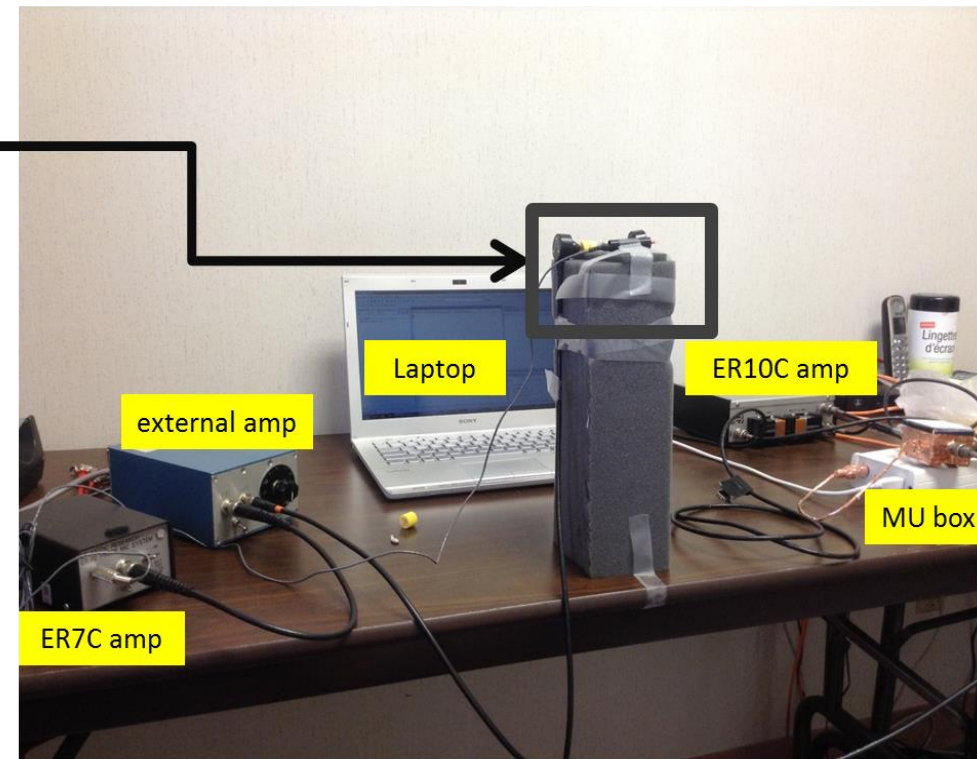
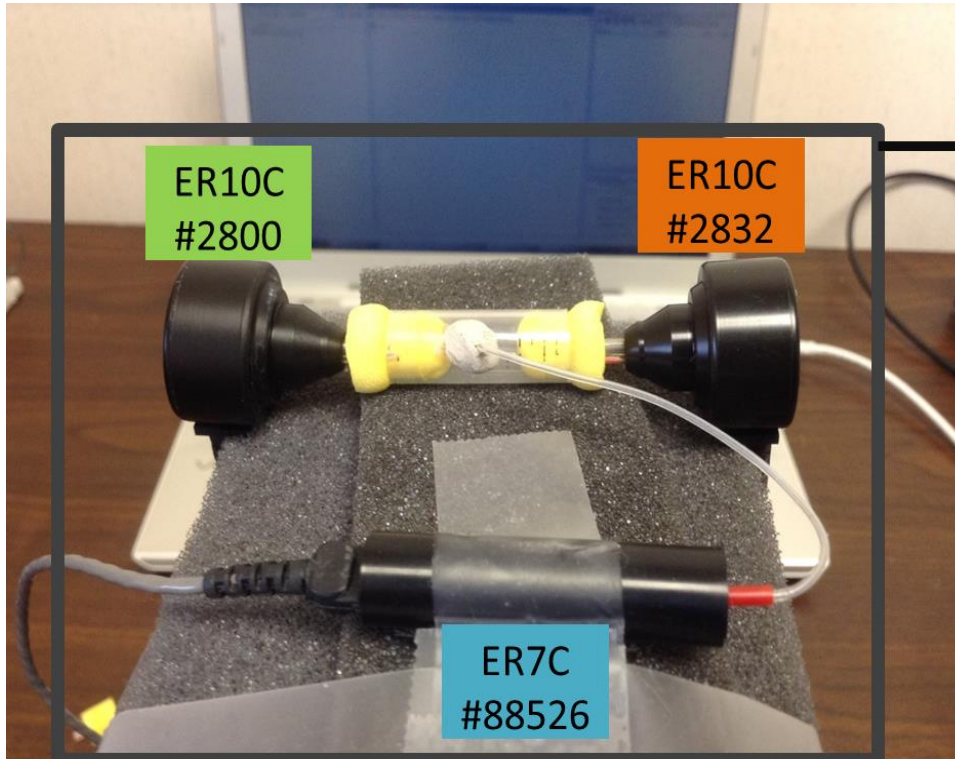
- Aluminum material for the middle tube holder part
- RTV is not used to block the holder's side hole, but some of black material seals the side hole fortunately
- Calibration passes up to 6kHz
- ER10C with 4 digits serial number



## New Bad ER10C

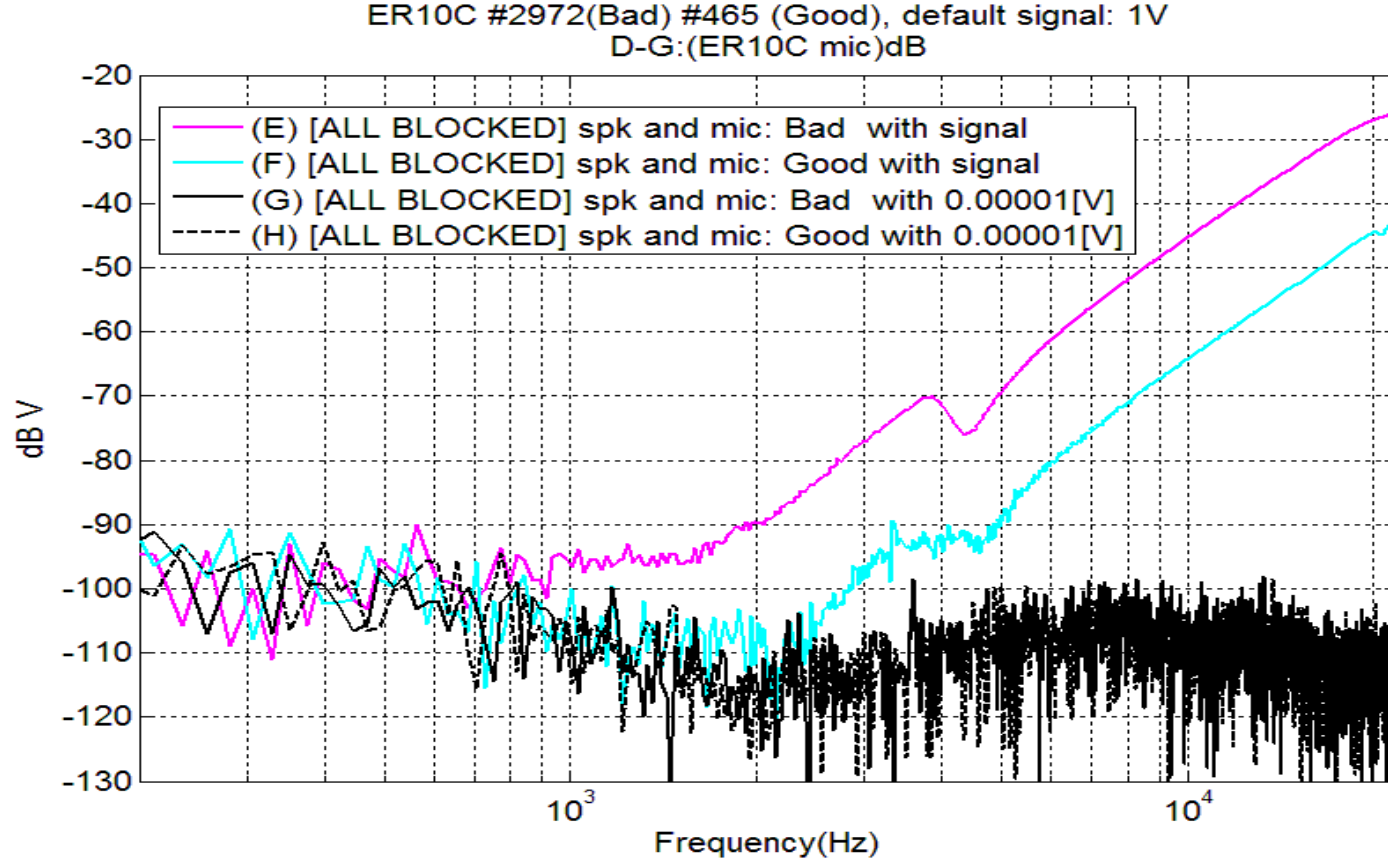
- Aluminum material for the middle tube holder part
- None of material seals the holder's side hole, a portion of the hole could be sealed randomly.
- Calibration totally fails or sometimes it passes but is unstable usually above 4kHz
- ER10C with 4 digits serial number

# Observation: Crosstalk in the system



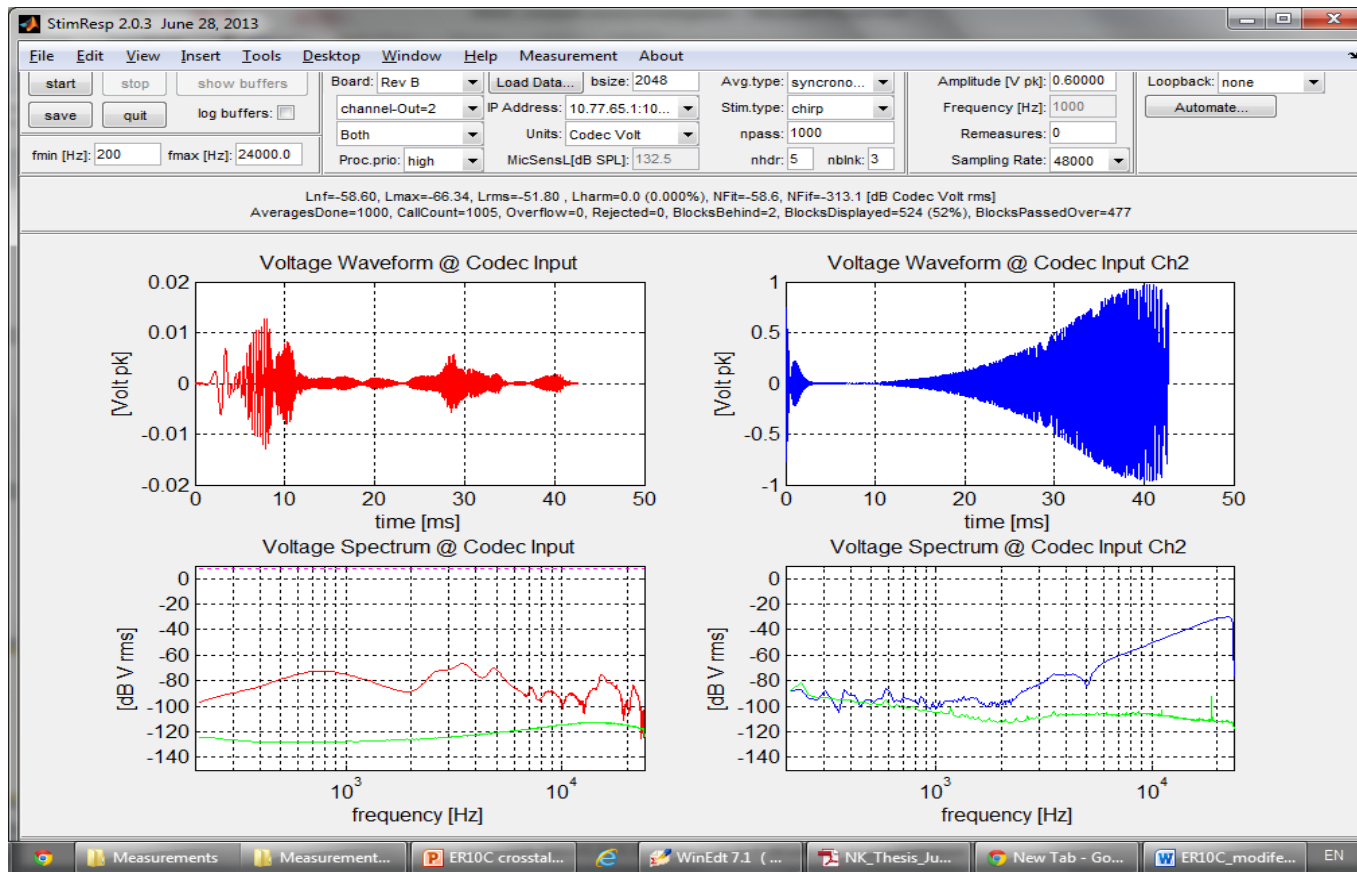
- We blocked the microphone hole on the ER10C foam tip to decouple the microphone sound path to the ER10C

# Observation: Crosstalk in the system



- We blocked the microphone hole on the ER10C foam tip to decouple the microphone sound path to the ER10C

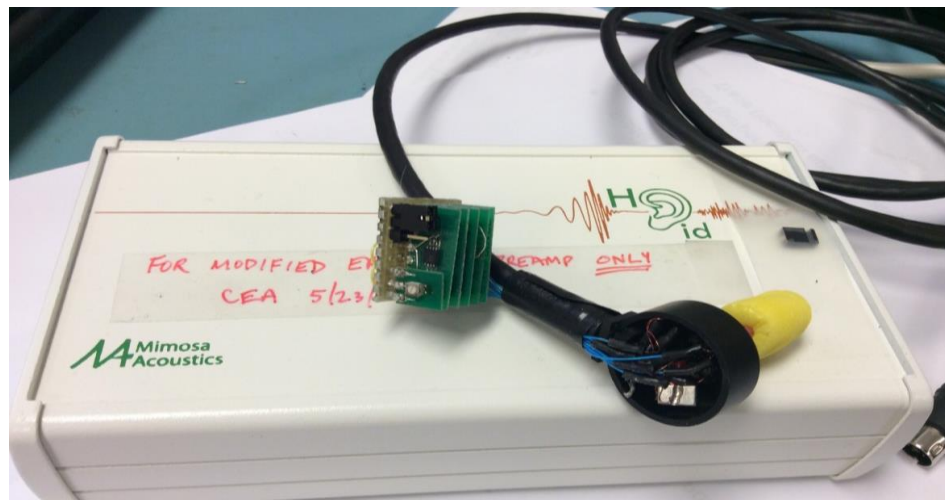
# Observation: Crosstalk in the system



- We blocked the microphone hole on the ER10C foam tip to decouple the microphone sound path to the ER10C
- Any signal that is shown on the right side of this figure (blue) can be assumed as the internal crosstalk (20dB/Oct)

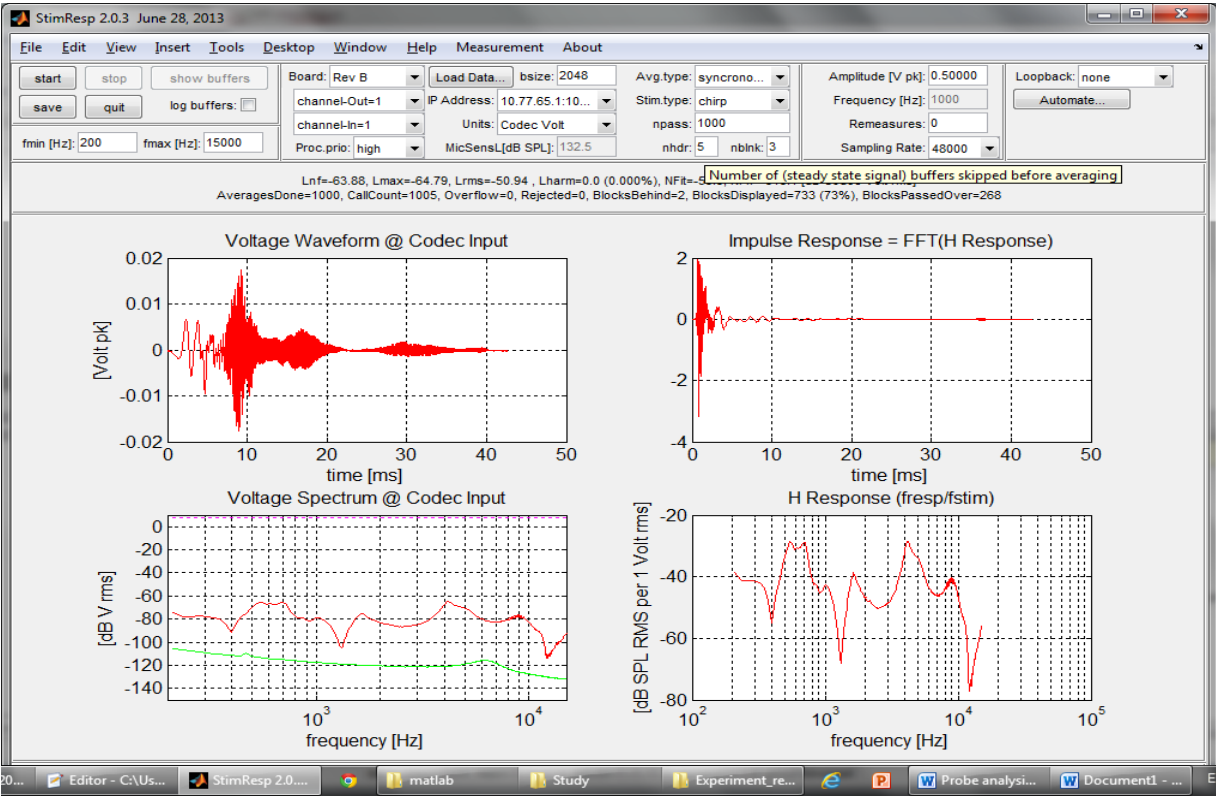
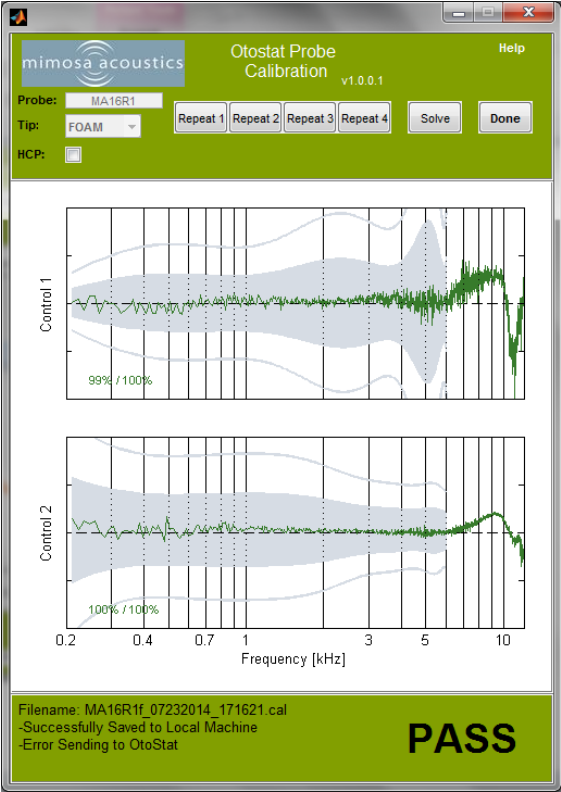
# Hypothesis, approach, and expectation

- The long wire attached to the ER10C probe contributes to the electrical crosstalk (capacitive coupling) in high frequency
- To lower the electrical crosstalk, we attached an external amplifier (close to the probe head)
- The available calibrating frequency range will be extended above 6 kHz

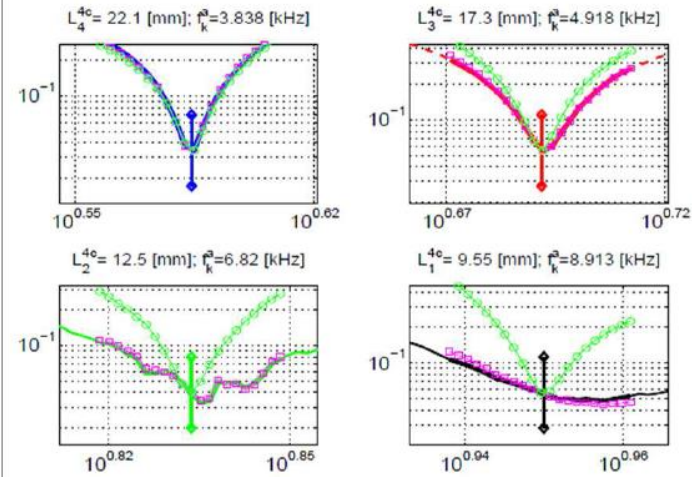
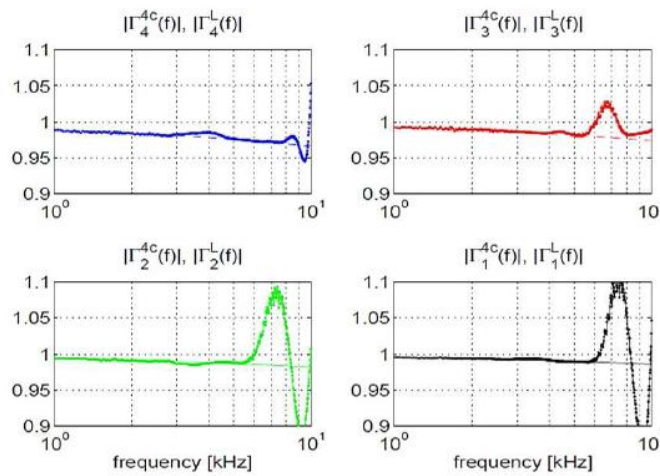


- Results: the calibration frequency range has been extended to 11kHz

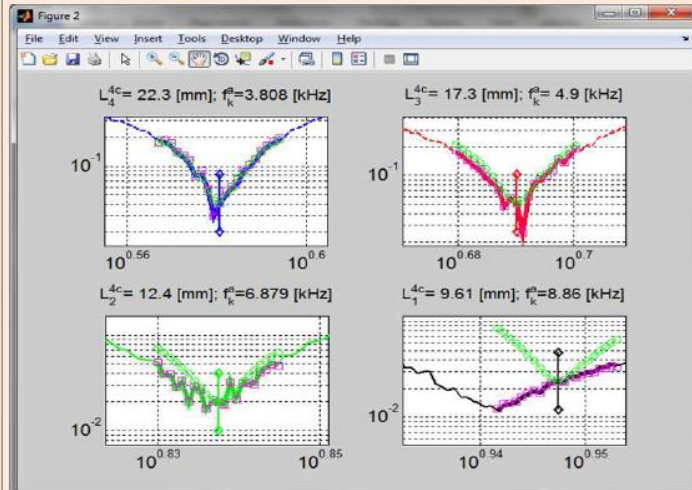
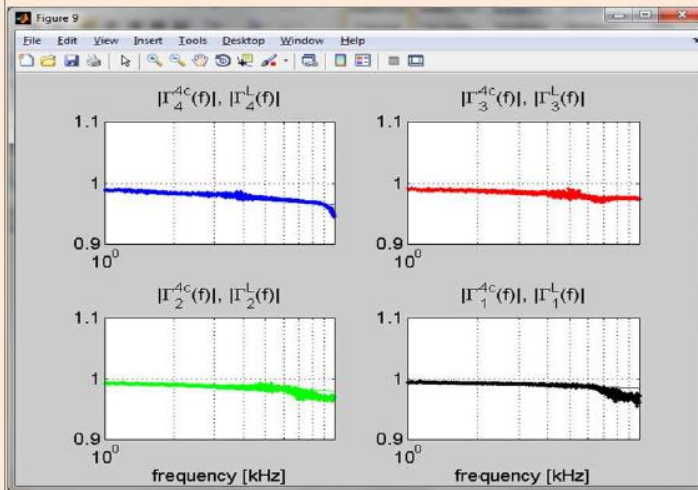
Crosstalk before/ after



B  
E  
F  
O  
R  
E



A  
F  
T  
E  
R



### |Gamma| in each cavity

- Theoretical, length based:  $\Gamma_k^L = e^{-2L_k \kappa(f)}$  (dashed line)
- Experimental:  $\Gamma_k^{4C} = (1 - Y_k^{4C}) / (1 + Y_k^{4C})$  (solid line) where  $Y_k^{4C} = U_s / P_k - Y_s$

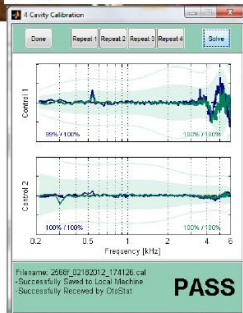
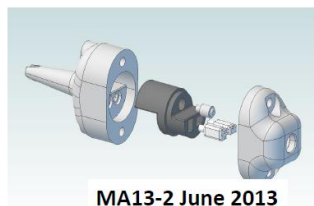
### Pressure null ( $c_0/4L_k \approx 45/L_k$ )

- Theoretical (green circle):  $P_k^L = P_s / (Y_s + Y_k^L)$  where  $Y_k^L = (1 - \Gamma_k^L) / (1 + \Gamma_k^L)$
- Experimental (pink circle)  $P_k^{4C} = P_s / (Y_s + Y_k^{4C})$

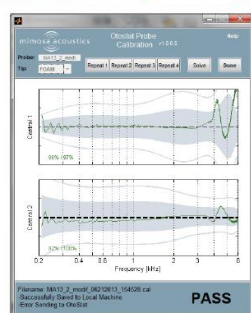


# Probe manufacturing and evaluation

## Learning from lots of trials (and errors)

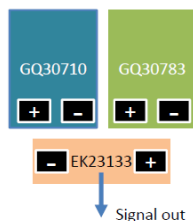


- These probes use TWFK 60173 receivers
- stable and continuous 4-cavity calibration pass

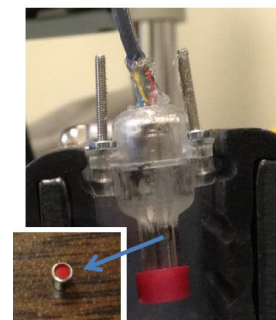


### MA 12-4

- Receivers
  - GQ 30710 (37ohm at 1kHz)
  - GQ 30783 (90ohm at 1kHz)
- Microphone
  - EK23133 (ER10C microphone)
- Physical structure looking at the electrical terminal end of the probe



### MA15-1

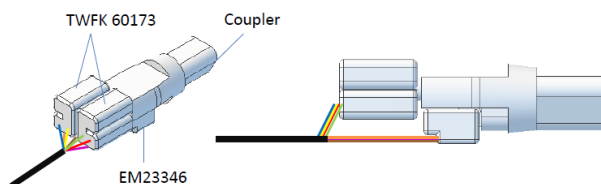


2.2k Ohm acoustic resistors (red) for receivers

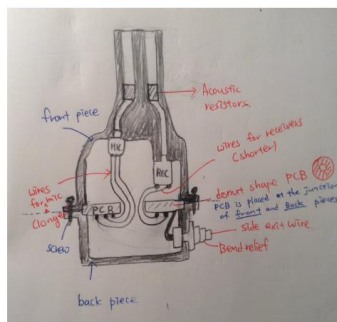
- Microphone (1): FG-23652
- Receivers (2) TWFK60173
- [2.2k ohm\(red\) acoustic resistors \(2\) for receivers \(Knowles BF 1921\)](#)
- Casing: 2 pieces, front and back
  - soft material compared to other MA15 series due to the manufacturing setting (not high resolution)
- Silicon glue, super glue, and liquid electrical tape

### MA12-1

- OUT: Knowles TWFK 60173 (2) receivers
- In: Knowles EM23346 (1) microphone
- ILO's coupler

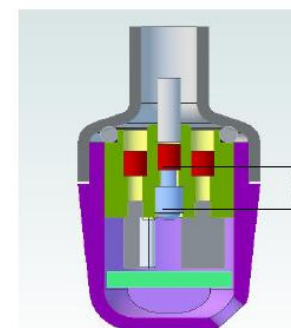


### Future direction



#### Key idea

- PCB board
- An amp is attached in probe's body (not shown in this picture)




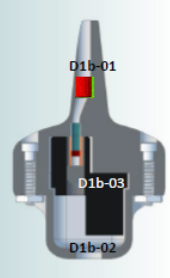
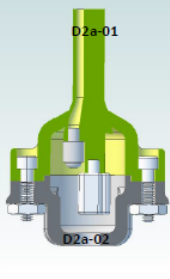
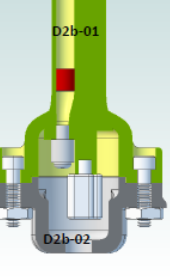
The receivers are generating sound behind the microphone

- But in MA16 current design, the position of the microphone is ahead of the receivers without considering the steel tube length.

# Sensitivity factors ( $\approx 0.8\text{cc}$ cavity)

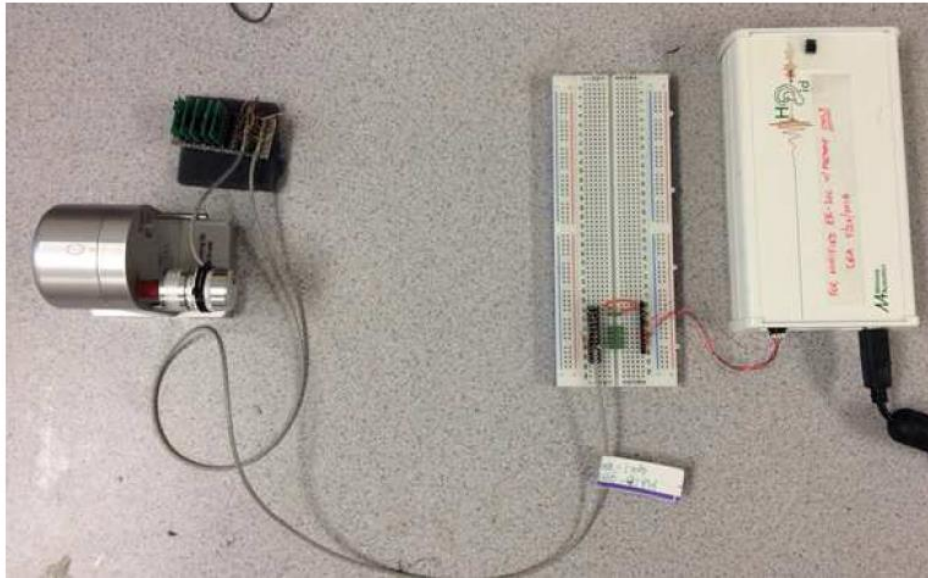
@ 996.1Hz	ER10C	MA12-3	ILO-TE	MA12-1	MA 12-2
Microphone	EK23133	EK23133	EM23346	EM23346	EM23346
Acoustic resistor	No	No	Yes	No	Yes
Foam tip	Yes	No	Yes	No	No
Cavity type	A	A	A	B	B
Microphone sensitivity [volt_peak/Pa]	0.1304	0.0301	0.1845	0.43829	0.1460
Receiver sensitivity [Pa/volt_peak]	2.1e4	1. e5	6 e3	456	7.5e4



Design Option	a: Without acoustic resistors	b: With acoustic resistors (■)
1	 <p><b>Code: D1a</b> Case (SLA, clear)</p> <ul style="list-style-type: none"> <li>Front part(tip): D1a-01</li> <li>Back part: D1a-02</li> </ul> <p><b>Middle part</b></p> <ul style="list-style-type: none"> <li>Black rubber: D1a-03</li> </ul>	 <p><b>Code: D1b</b> Case (SLA, clear)</p> <ul style="list-style-type: none"> <li>Front part(tip): D1b-01</li> <li>Back part: D1b-02</li> </ul> <p><b>Middle part</b></p> <ul style="list-style-type: none"> <li>Black rubber: D1b-03</li> </ul>
2	<p><b>MA15-3</b></p>  <p><b>Code: D2a</b> Case (SLA, clear)</p> <ul style="list-style-type: none"> <li>Front part(tip): D2a-01</li> <li>Back part: D2a-02</li> </ul> <p><b>No middle part</b></p>	<p><b>MA15-1,2,4,5</b></p>  <p><b>Code: D2b</b> Case (SLA, clear)</p> <ul style="list-style-type: none"> <li>Front part(tip): D2b-01</li> <li>Back part: D2b-02</li> </ul> <p><b>No middle part</b></p>

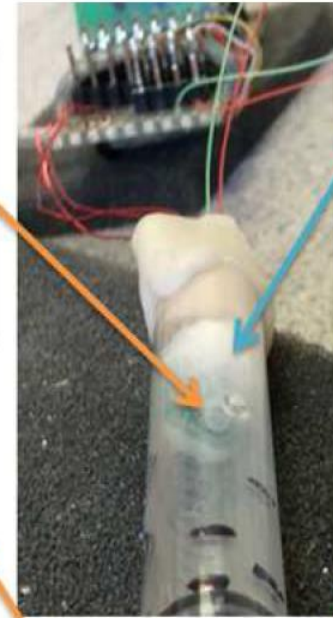


## ■ MA16 & MA17 simulator



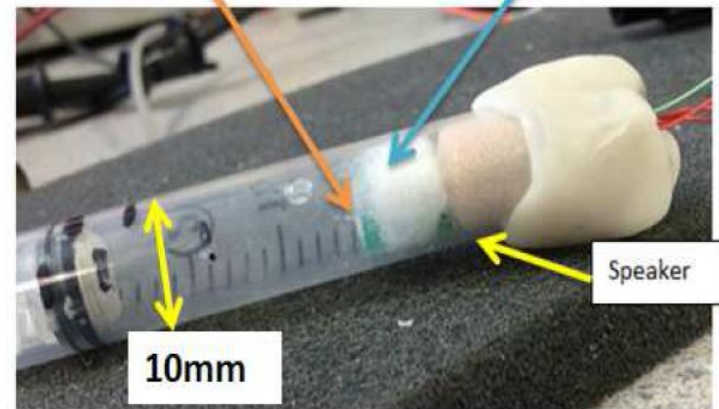
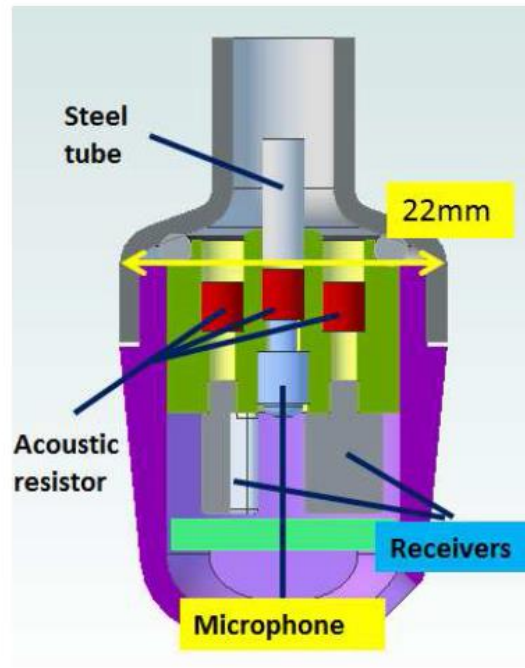
Microphone rubber tube

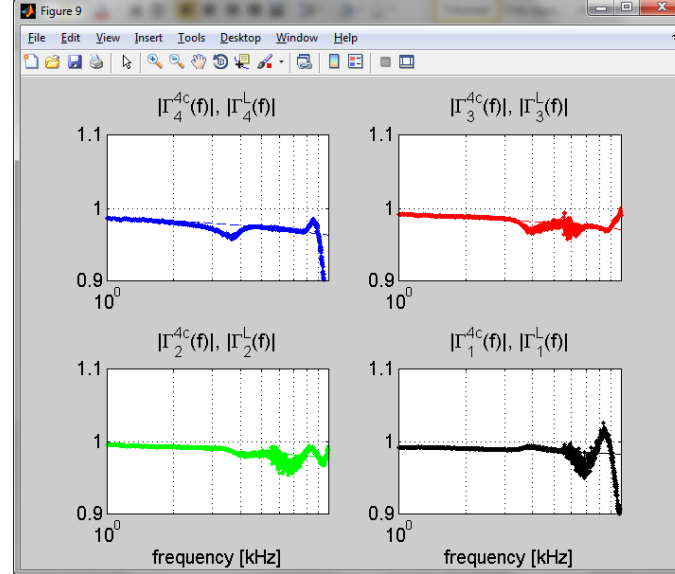
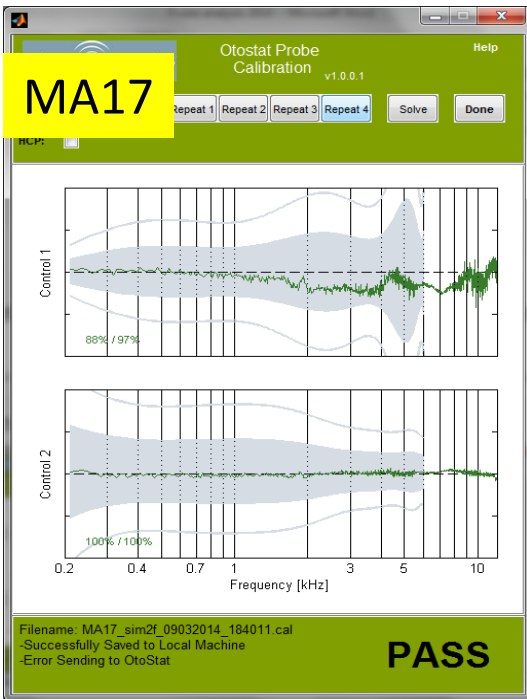
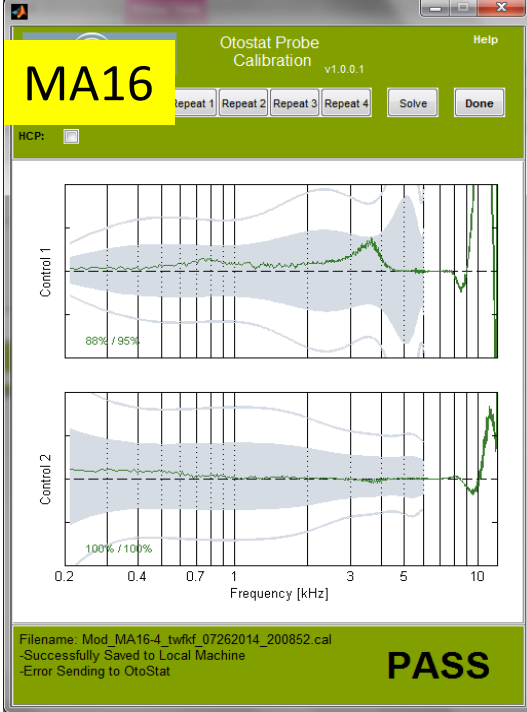
- Nearly centered
- 1 or 2 mm ahead of the small square, inner board



Cotton piece

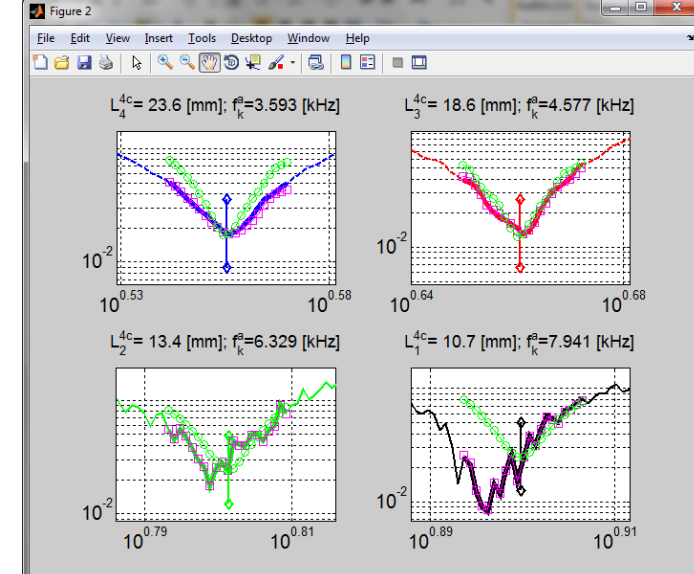
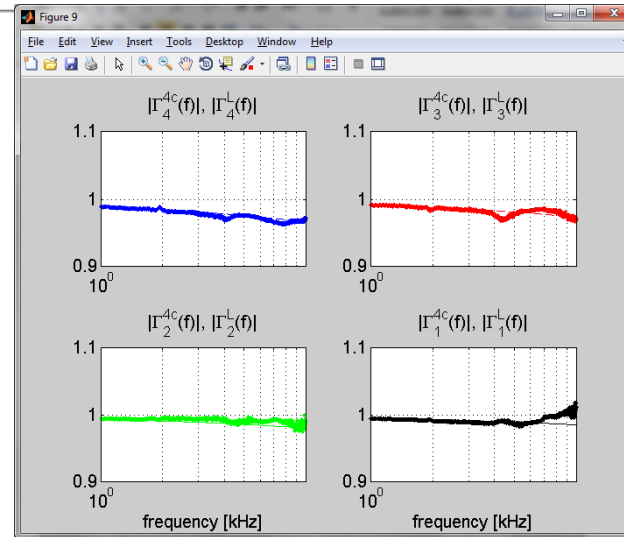
- Remove speaker distortions
- Packing the space between the speaker and microphone to support to make the cylindrical structure (cavity)





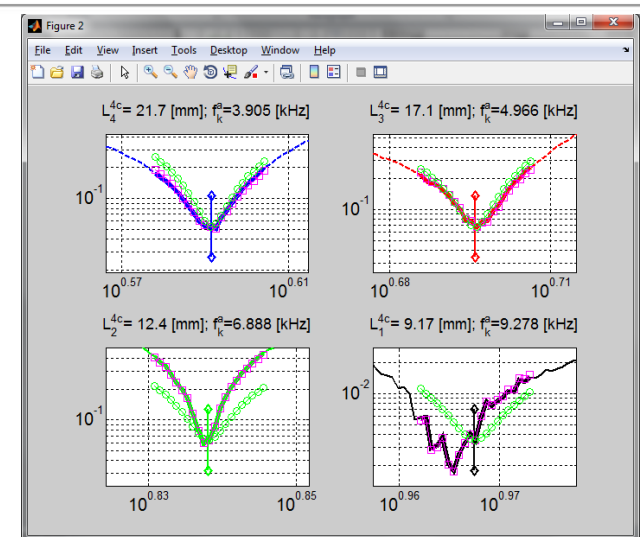
**|Gamma| in each cavity**

- Theoretical, length based:**  $\Gamma_k^L = e^{-2L_k \kappa(f)}$  (dashed line)
- Experimental:**  $\Gamma_k^{4C} = (1 - Y_k^{4C}) / (1 + Y_k^{4C})$  (solid line) where  $Y_k^{4C} = U_s / P_k - Y_s$



**Pressure null ( $c_0/4L_k \approx 45/L_k$ )**

- Theoretical (green circle):**  
 $P_k^L = P_s / (Y_s + Y_k^L)$   
 where  $Y_k^L = (1 - \Gamma_k^L) / (1 + \Gamma_k^L)$
- Experimental (pink circle)**  
 $P_k^{4C} = P_s / (Y_s + Y_k^{4C})$



# Probe evaluation

- 1. Frequency responses of both microphone and speaker should be as flat as possible**
  - especially within the frequency range of human hearing (ideally up to 20kHz for the microphone and up to 16kHz for the speaker)
- 2. Thevenin parameters must be stable over time**
  - This can be evaluated via source calibration (i.e., 4 cavity calibration, Allen (1986))
- 3. Output levels for loudspeakers should be higher**
  - especially for measuring hearing impaired ears. (i.e., 85dB SPL desirable)
- 4. Dynamic range as large as possible**
  - Dynamic range is defined as the difference between the first harmonic level and the total harmonic level at each frequency (i.e., 50-60dB is acceptable)

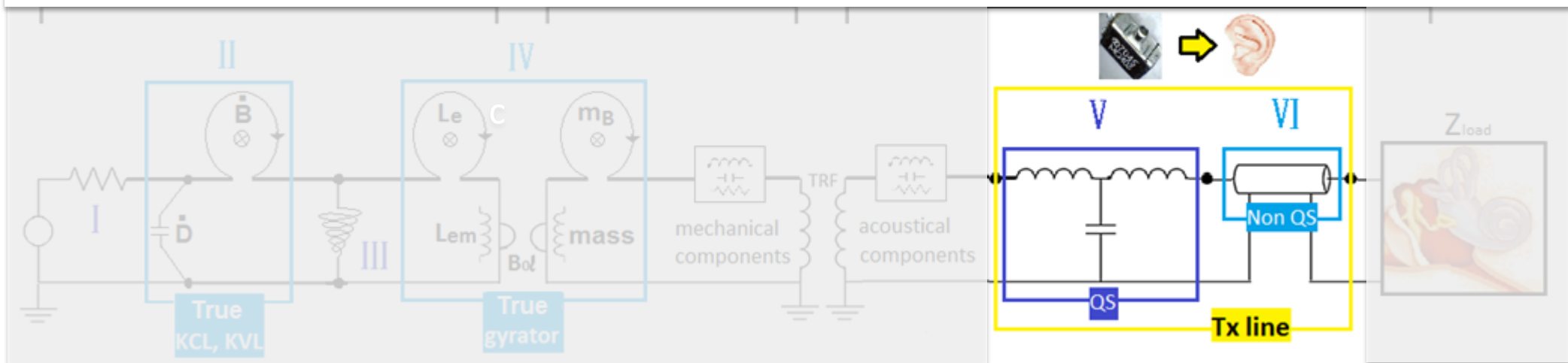
- 5. Linearity superior to current probes**
  - Dynamic range should be linear across the frequency range of interest
- 6. Impulse response should be short and exact**
  - The duration of impulse ringing should be less than 1 ms. This result is critical to TEOAE measurement
- 7. Crosstalk issues including all noise sources must be addressed - microphone, loudspeaker...**
- 8. Good seal and stability in the ear canal**
  - This needs good earplug design to fit a range of adult ear-canal sizes and shapes easily
- 9. The size of the probe is an especially critical factor in the clinic for measurements of infant ears, due to their very small ear canals.**

These must take into account in the probe design !!

# Sub conclusion from Experimental part

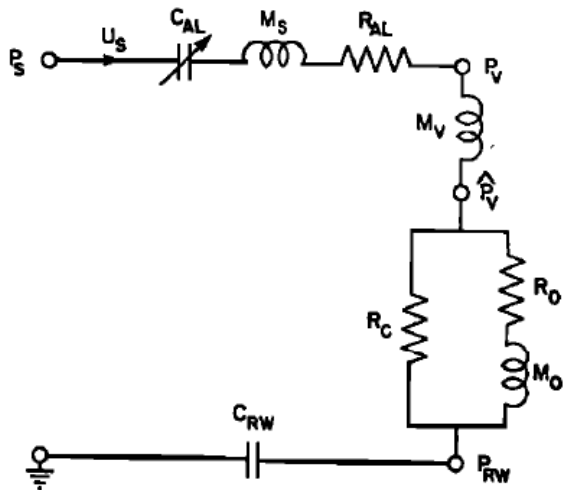
- We have solved the crosstalk problem in the ER10C which has kept users from calibrating the probe above 6 kHz
  - Now the system can pass 4C calibration above 10 kHz
- The MA16 and MA17, prototype probes, have comparable performance characteristics
- This study shows that crosstalk may be a general problem for OAE hearing probe devices which needs to be carefully addressed in the design process

# Quasi-static delay



- When we deal with a physical system, such as ear canal, transmission line representation is simpler and intuitive way to model the system
  - ✓ Lumped element can mimic the system almost identically, but needs more element than a single transmission line
  - ✓ We don't have to worry about the band limitation of the system
  - ✓ We need a transmission line to accurately model for our system

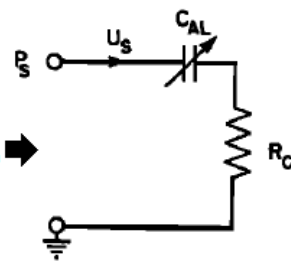




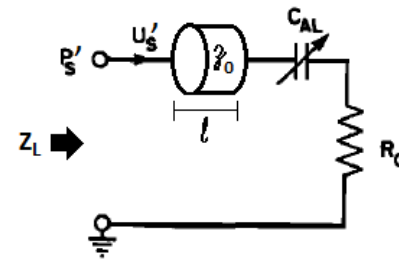
(a) Original electrical network representation of the impedance of the stapes and cochlea by Lynch et al (1982)



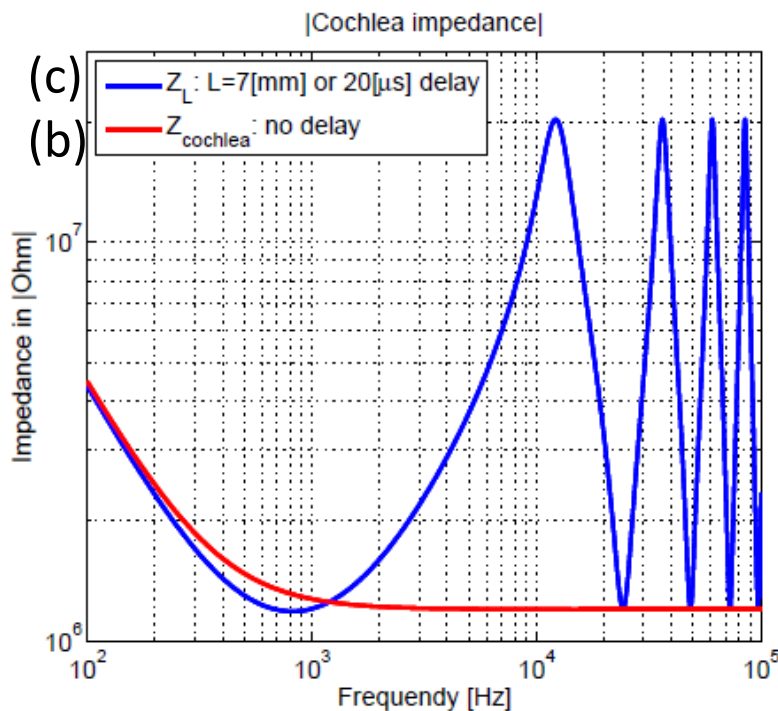
$Z_{cochlea}$



(b) Simplified version of (a) with minimum lumped circuit elements without delay line



(c) Delay (transmission line) is added to (b)



- $Z = \frac{1+\Gamma}{1-\Gamma}$  where the reflectance  $\Gamma = e^{-s\frac{l}{c}}$ .
- $\tau = \frac{l}{c}$  represents a pure delay
- The  $l, c$  stand for the length of the ear canal and speed of sound
- When  $\Gamma = \pm 1$ , poles and zeros appear in impedance domain (magnitude), respectively
- The reflection of the wave relates to the standing wave.

# CONCLUSIONS AND EXPECTED CONTRIBUTIONS

## 1. The uniqueness of our BAR model

- Extends anti-reciprocal networks using a gyrator
- Includes a semi-inductor in the network
- Represents non quasi-static networks by means of transmission line

## 2. The $Z_{\text{mot}}$ is not a physically realizable PR impedance supported by

- PR property, it's not a driving point impedance
- A simplified electro-mechanic model simulation

(Physical explanation about the negative  $Z_{\text{mot}}$  real parts :Eddy-currents loss)

## 3. A generalization of the ABCD matrix cascading method

- Characterized by the Möbius transformation
- Found isomorphic relation between two methods

4. **In-depth investigation of the BAR's operational principles**
  - Reinterpreting the gyrator including the AC magnetic flux along with DC flux
  - Apply and investigate the classic theories to the specific BAR case, such as KCL, KVL, and the diffusive wave equation dynamic (or non-QS) terms
5. **Providing technical understanding of not only ER10C system but also hearing measurement devices in general**

### **Summary of contributions:**

**This analysis puts “the anti-reciprocal electro-magnetic” transducer’s theory and application on a firm basis**

## 's' or 'ω'

- Proper frequency domains for signals and systems
  - Signals (i.e.,  $\phi$ ,  $I$ ) and systems (i.e., power and impedance)
- System: Laplace frequency  $s = \sigma + j\omega$ 
  - Indicate Positive-Real (PR) system
    - strictly non negative on the right half of the Laplace plane (passive condition )
  - In Laplace frequency plane, the abscissa (x-axis) is for a real part ( $\sigma$  referring to any loss in a system) while the ordinate (y-axis) is for an imaginary part ( $j\omega$  where  $\omega$  is an angular frequency or a Fourier frequency)
- Signals: They do not need to obey the PR property
  - Angular Fourier frequency  $\omega$  is used
  - i.e.,  $\phi(\omega)$  and  $I(\omega)$  are complex quantities
- For example, one can use Fourier transform to convert a voltage in the time domain to a voltage in the frequency domain. But to convert power from one domain to the other, the Laplace transform must be applied

# False in general vs. True always

for the bigger task.

But there is the danger in this process that before we get to see the complete story, the incomplete truths learned on the way may become ingrained and taken as the whole truth—that what is true and what is only sometimes true will become confused. So we give in Table 15-1 a summary of the important formulas we have covered, separating those which are true in general from those which are true for statics, but false for dynamics. This summary also shows, in part, where we are going, since as we treat dynamics we will be developing in detail what we must just state here without proof.

It may be useful to make a few remarks about the table. First, you should notice that the equations we started with are the *true* equations—we have not misled you there. The electromagnetic force (often called the *Lorentz force*  $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$  is *true*. It is only Coulomb's law that is false, to be used only for statics. The four Maxwell equations for  $\mathbf{E}$  and  $\mathbf{B}$  are also true. The equations we took for statics are false, of course, because we left off all terms with time derivatives.

The Feynman Lectures  
on Physics  
Volume II,  
electromagnetism and  
matter

- ***But there is the danger in this process that before we get to see the complete story, the incomplete truths learned on the way may become ingrained and taken as the whole truth—that what is true and what is only sometimes true will become confused.....***

Table 15-1

FALSE IN GENERAL (true only for statics)	TRUE ALWAYS
$F = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r^2}$ <p>(Coulomb's law)</p>	$F = q(E + v \times B)$ <p>(Lorentz force)</p>
$\nabla \times E = 0$	$\nabla \cdot E = \frac{\rho}{\epsilon_0}$ <p>(Gauss' law)</p>
$E = -\nabla\phi$	$\nabla \times E = -\frac{\partial B}{\partial t}$ <p>(Faraday's law)</p>
$E(1) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(2)e_{12}}{r_{12}^2} dV_2$	$E = -\nabla\phi - \frac{\partial A}{\partial t}$ <p><math>E(\phi, A)</math> (electric field)</p>
<p>For conductors, <math>E = 0</math>, <math>\phi = \text{constant}</math>, <math>Q = CV</math></p>	<p>In a conductor, <math>E</math> makes currents.</p>
$c^2 \nabla \times B = \frac{j}{\epsilon_0}$ <p>(Ampere's law)</p>	$\nabla \cdot B = 0$ <p>(No magnetic charges)</p>
$B(1) = \frac{1}{4\pi\epsilon_0 c^2} \int \frac{j(2) \times e_{12}}{r_{12}^2} dV_2$	$B = \nabla \times A$ <p><math>B(A)</math> (magnetic field)</p>
$\nabla^2 \phi = -\frac{\rho}{\epsilon_0}$ <p>(Poisson's equation)</p>	$\nabla^2 A - \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} = -\frac{j}{\epsilon_0 c^2}$ <p>and</p>
$\nabla^2 A = -\frac{j}{\epsilon_0 c^2}$	$c^2 \nabla \cdot A + \frac{\partial \phi}{\partial t} = 0$ <p>with</p>
<p>with</p> $\nabla \cdot A = 0$	<p><math>\phi(1, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(2, t')}{r_{12}} dV_2</math></p> <p>and</p> $A(1, t) = \frac{1}{4\pi\epsilon_0 c^2} \int \frac{j(2, t')}{r_{12}} dV_2$ <p>with</p> $t' = t - \frac{r_{12}}{c}$ <p>retarded potentials Einstein causality</p>
$\phi(1) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(2)}{r_{12}} dV_2$	$U = \int \left( \frac{\epsilon_0}{2} E \cdot E + \frac{\epsilon_0 c^2}{2} B \cdot B \right) dV$ <p>Energy statics vs Energy dynamics</p>
$A(1) = \frac{1}{4\pi\epsilon_0 c^2} \int \frac{j(2)}{r_{12}} dV_2$	<p><math>\rightarrow \text{D.F.} = \frac{1}{2} \text{D.R.}</math></p>
$U = \frac{1}{2} \int \rho\phi dV + \frac{1}{2} \int j \cdot A dV$	

- The Table [15-1](#) separates those which are true in general from those which are true for statics, but false for dynamics
- The equations we started with are the *true* equations.
  - The electromagnetic force (often called the *Lorentz force*)  $F=q(E+v \times B)$  is *true*. It is only Coulomb's law that is false, to be used only for statics.
  - The four Maxwell equations for E and B are also true. The equations we took for statics are false, of course, because we left off all terms with time derivatives (**true KCL, KVL**)

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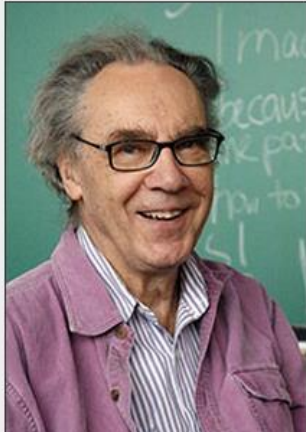
## Lecture Notes

Selected lecture notes are provided below.

COURSE HOME	LEC #	LECTURE NOTES
SYLLABUS	7	Capacitance of Spheres ( <a href="#">PDF</a> )
CALENDAR	8	Dielectrics and Polarization ( <a href="#">PDF</a> )
LECTURE NOTES <	11	Build Your Own Electric Motor - Have Fun, and Earn Extra Credit ( <a href="#">PDF</a> )
	13	Circular motion of Protons and Electrons - Particle Accelerators ( <a href="#">PDF</a> )
	15	Mid-term Evaluation Form ( <a href="#">PDF</a> )
ASSIGNMENTS	16	Non-conservative Fields - Do Not Trust Your Intuition ( <a href="#">PDF</a> )
EXAMS	20	Faraday's Law - Most Physics College Books have it WRONG! ( <a href="#">PDF</a> ) Driven L-R Circuits ( <a href="#">PDF</a> )

## Faculty

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 Professor of Physics, *Emeritus*



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 Cambridge, MA 02139

- It will be great, if this point can be emphasized at the beginning not to mislead students
- A different view to see one problem. But it is important to have view of diversity 😊

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**THANK YOU 😊**

